Costas sequences with high PSLR

A.NAGAJYOTHI
Research Scholar, Department of Electronics & Communication Engineering, Andhra University, Visakhapatnam

PROF. K.RAJA RAJESWARI
Professor, Department of Electronics& Communication Engineering, Andhra University, Visakhapatnam

Abstract-- **The normal Costas frequencies sequence is very well known and important signal in pulse compression Radar. In this paper we present two algorithms to modify Costas sequence by arranging the frequencies of Costas signal in time, first using Welch Costas array and second using Golomb Ruler. These methods enable us to control side-lobes and to improve Doppler frequency resolution of Ambiguity Function (AF).**

At first we present the principle of these methods: Welch Costas array and Golomb Ruler. Then we apply these two methods to normal Costas signal and calculate the AF ,ACF .We take the best sequence by calculating the PSLR for all the codes obtained by Welch and Golomb Ruler methods.

The results of comparison have shown that considerable reduction of side-lobes of AF is achieved by using these two methods, and consequently an improvement of AF is obtained.

Keywords -AF: Ambiguity Function, Costas signal, ACF: Autocorrelation Function, Golomb rulers, Welch sequence, PSLR: Peak Side Lobe Ratio.

1. INTRODUCTION

The essential part of improving radar system performance is to develop new radar signal. Consideration is given to the effect of delay resolution, Doppler resolution, noise immunity, and intentional and unintentional interference suppression. The aforesaid requirements have caused to develop these signals assisted by modern signal processing systems. In general for good detection most radars seek to transmit long-duration pulses to achieve high energy, since transmitters are typically operated near their peak power limitation. On the other hand, for good range measurement accuracy, radar needs short pulses. These divergent of the needs of long pulses for detection and short pulses for range resolution in measurements prevented early radars from simultaneously performing both functions. Fortunately, in the late 1950s and early 1960s a new concept was developed whereby both needs could be met. The concept is called **Pulse Compression** [1,6,7,11,12].

Pulse compression allows radar to utilize a long pulse to achieve large radiated energy, but simultaneously to obtain the range resolution of a short pulse. It accomplishes this by employing frequency or phase modulation to widen the signal bandwidth. The received signal is processed in a matched filter that compresses the long pulse duration 1/B, where B is the modulated-pulse spectral bandwidth.

Pulse compression is a method for achieving most of the benefits of a short pulse while keeping within the practical constraints of the peak-power limitation. Pulse compression is achieved by modulating the transmitted pulse and then correlating the received signal with the transmitted pulse. Pulse compression also known as pulse coding, is a signal processing technique designed to maximize the sensitivity and resolution of radar system. Radar signal designers had been continuously putting their efforts to achieve suitable codes for the optimum performance of radar. The researchers develop many Pulse compression radar signals assisted by modern signal processing systems. Consequently, signals in different shapes have been presented like **Phase coded signals** such as Barker code, Frank code, P1, P2, P3, Px codes, as well as m-sequence code etc., and **Frequency coded signals** such as Costas signal[3,4,5]. Each of these signals has advantages and disadvantages. The most important one in the frequency coding is Costas signal.

Costas sequences are generally used in the design of frequency-coded waveform, which ensure high delay-Doppler Resolution. An important property of Costas sequence is that a sequence of length N when used in the radar signal design would yield an Ambiguity Function(AF) with side-lobes of maximum height 1/N times of its main-lobe height[11,12].

In radar scenario, no waveform is optimum for target resolution in general. On the other hand, an optimum ambiguity surface should be of a sharp central spike surrounded by a clear area with no volume, then when the bulk of the volume pushed away from the central peak, then the interference can be avoided[1,2,4].In this paper we suggested two algorithms to arrange the frequencies in time to enable us to control the side-lobes by using Golomb ruler and Welch Costas array or simply Costas array.

Costas pulse T consists of N sub-pulses; each sub-pulse has different frequency modulation as shown in Fig 1.a shows Welch Costas sequence for N=11 and Fig 1.b shows Golomb Ruler for N=11. Each frequency is chosen from a series of frequencies within the bandwidth B. We have N frequencies each frequency is multiple of $\Delta f = {}^{1}/_{t_p}$ and pulse width of each sub-pulse is given by $t_p = {}^{T}/_{N}$. Costas has suggested algorithm to arrange the frequencies to enable us to control the side-lobes in such a way that these side-lobes will not exceed 1/N. Then, the biggest side-lobes in AF is 1/N of its value at the main-lobe Fig.2.a and Fig.2.b Costas signal has a delay resolution of $1/N^2$ and the Doppler resolution of 1/T and because of using a matched filter, The received signal has noise immunity. However, Costas pulse is not an optimum signal [2,4].

2. CONSTRUCTION OF COSTAS SEQUENCE BY APPLYING WELCH AND GOLOMB RULER.

2.1. WELCH CODING

Following procedure is used to find the Costas signals for a sequence of length N. Let p be an odd, prime number. Then N = p-1. Let g be the primitive root of p. A primitive root of p is defined as the value g such that the sequence of powers $g_1, g_2, g_3, \ldots g_{p-1}$ modulo p generates every integer from 1 to p-1. A primitive root of p is NOT the same as a prime factor of p or N, though a primitive root may be a factor of N. Costas found ideal frequency-time sequences up to 12. For instance, the primitive root of 41 is 6, which is neither a factor of p = 41 or of N = 40. A compilation of all known sequences for N up to 360 is given by Golomb and Taylor.
Once we have the sequence, we can then construct a difference matrix, which will allow the construction of the resultant Side-lobe matrix, which will provide insight into the form of the ambiguity function.

TABLE I. Welch Costas sequence up to N=11.

Order	Length	Marks
1	2	1 2
2	3	1 3 2 2 3 1
3	4	1 2 4 3 1 3 4 2
4	5	2 1 5 3 4 4 2 5 1 2
5	6	1 3 2 6 4 5 1 5 4 6 2 3
6	8	2 6 3 8 7 5 1 4
7	9	1 3 7 4 9 8 6 2 5 5 2 6 8 9 4 7 3 1 6 4 1 2 9 3 5 8 7 7 8 5 3 9 2 1 4 6
8	10	1 2 4 8 5 10 9 7 3 6 1 6 3 7 9 10 5 8 4 2 1 7 5 2 3 10 4 6 9 8 1 8 9 6 4 10 3 2 5 7
9	11	1 3 7 2 5 11 10 8 4 9 6 5 9 7 8 1 11 6 2 4 3 10 6 9 4 8 10 11 5 2 7 3 1 10 3 4 2 6 11 1 8 7 9 5

2.2. GOLOMB CODING

A **Golomb ruler** is a set of marks at integer positions along an imaginary ruler such that no two pairs of marks are the same distance apart. The number of marks on the ruler is its *order*, and the largest distance between two of its marks is its *length*. Translation and reflection of a Golomb ruler are considered trivial, so the smallest mark is customarily put at 0 and the next mark at the smaller of its two possible values.

The Golomb ruler was named for Solomon W. Golomb and discovered independently by Sidon and Babcock[6].

There is no requirement that a Golomb ruler be able to measure *all* distances up to its length, but if it does, it is called a *perfect* Golomb ruler.

It has been proven that no perfect Golomb ruler exists for five or more marks. A Golomb ruler is *optimal* if no shorter Golomb ruler of the same order exists. Creating Golomb rulers is easy, but finding the optimal Golomb rulers for a specified order is computationally very challenging.

The Table.2. contains all known optimal Golomb rulers.

TABLE II. Golomb sequence up to N=11

Order	Length	marks
1	0	0
2	1	0 1
3	3	0 1 3
4	6	0 1 4 6
5	11	0 1 4 9 11 0 2 7 8 11
6	17	0 1 4 10 12 17 0 1 4 10 15 17 0 1 8 11 13 17 0 1 8 12 14 17
7	25	0 1 4 10 18 23 25 0 1 7 11 20 23 25 0 1 11 16 19 23 25 0 2 3 10 16 21 25 0 2 7 13 21 22 25
8	34	0 1 4 9 15 22 32 34
9	44	0 1 5 12 25 27 35 41 44
10	55	0 1 6 10 23 26 34 41 53 55
11	72	0 1 4 13 28 33 47 54 64 70 72 0 1 9 19 24 31 52 56 58 69 72

3. FORMATION OF DIFFERENCE TRIANGLE

In Costas code the discrete modulation of the carrier frequency is used to encode the transmitted pulse. Costas signal of order N is a burst of ‘N’ continuous pulses each of duration ‘τ’ and each at a frequency i.e., the signal consists of N frequencies being chosen from a set $\{f_1, f_2, f_3, \ldots, f_n\}$ of available frequencies for transmission at each set $\{\tau_1, \tau_2, \tau_3, \ldots, \tau_n\}$ of consecutive time intervals where $\tau_1 = \tau_2 = \tau_3 = \ldots = \tau_n = \tau$.

In an N x N frequency time array, Costas has given simple solution to select group patterns.

The difference triangle formed from the ordered sequence shall have no repeated term in any row. Taking first row differences between adjacent numbers, all differences in this row must be unique.

For the second row taking difference between next adjacent numbers in the same first row. This row must also be free of repeated values and so forth.

TABLE II. Difference triangle for Golomb sequence N=11

0	1	9	19	24	31	52	56	58	69	72
1	8	10	5	7	21	4	2	11	3	
9	18	15	12	28	25	6	13	14		
19	23	22	33	32	27	17	16			
24	30	43	37	34	38	20				
31	51	47	39	45	41					
52	55	49	50	48						
56	57	60	53							
58	68	63								
69	71									
72										

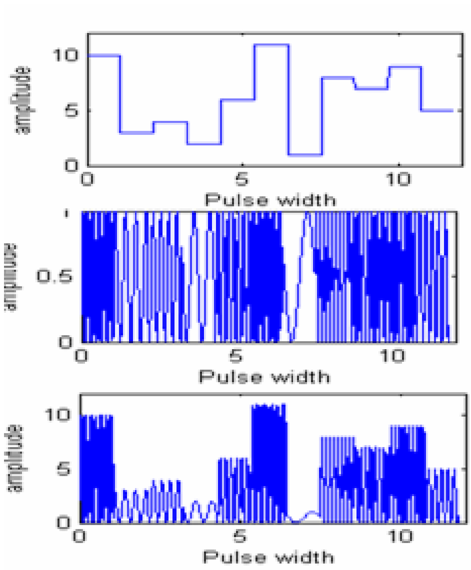

Figure. 1.a. Welch Costas sequence N=11[10 3 4 2 6 11 1 8 7 9 5]

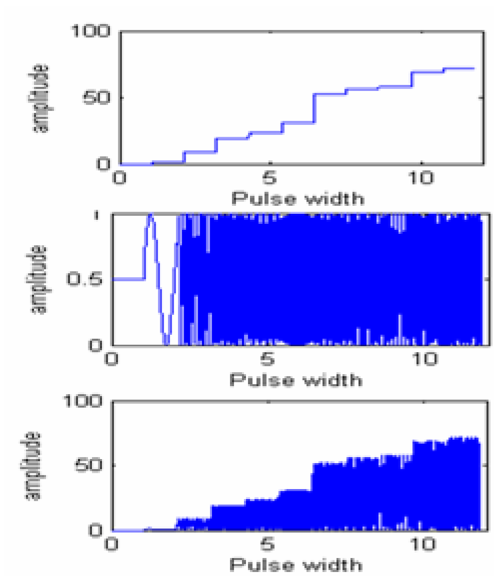

Figure 1.b. Golomb frequency coding N=11[10 1 9 19 24 31 52 56 58 69 72]

4. PRINCIPLE OF AMBIGUITY FUNCTION

An ideal AF requires narrow central peak and low noise floor. This implies that the ratio between main peak and side lobe level peak should be as high as possible. In view of invariance property of AF, energy removed / reduced at one place has to appear at another place. Thus it is required that most of the energy is contained in the main lobe and as less as possible in the side-lobes[2,3,4].

In radar signal processing, the problem is the estimation of the range and the radial velocity of a moving target where the range is proportional to round trip travel time i.e the delay time of the radar signal while the radial velocity is proportional to the Doppler frequency shift. If radar signal estimation is maximized when the receiver filter is matched to the transmit waveform. The output of the matched filter is equal to the cross section between the received signal and the transmitted signal.

Figures 2.b,3.b,4.b,5.b,6.b represents the AF plot in which X-axis is Range axis, Y-axis is Doppler axis and Z-Axis is output of the Matched filter[1,2].

When the Doppler axis is made zero the there are only two axis, one is Range and second is Matched filter output. These two represents the ACF.

This means that ACF is AF at Doppler zero which are shown in figures 2.a,3.a,4.a,5.a,6.a,7.a

5. RESULTS.

Application of using Golomb rulers and Welch Costas in frequency coded signals.

5.1. Peak Side Lobe ratio.

One of the most commonly used measuring performances in radar is Peak Side Lobe ratio (PSLR). The PSLR is given by the ratio of peak side lobe amplitude to the main lobe peak amplitude and is generally expressed in decibels[12].

$$PSLR = 20\log_{10}\left[\max\left\{\frac{r(i)}{r(o)}\right\}\right] \quad i \neq 0 \qquad (1)$$

5.2 FREQUENCY CODED: WELCH SEQUENCE.

Here in this paper we have chosen N=11. And for N=11 we have four different sequence which can be used as Costas pulse compression codes as they satisfy the difference triangle formation and as said in section-3 ,a sequence can be used as Costas pulse compression when they satisfy difference triangle formation. For N=11 {1 3 7 2 5 11 10 8 4 9 6, 5 9 7 8 1 11 6 2 4 3 10, 6 9 4 8 10 11 5 2 7 3 1, 10 3 4 2 6 11 1 8 7 9 5} .

These are the four sequences in which we have taken one code which as best PSLR. We found that N=11 {10 3 4 2 6 11 1 8 7 9 5} has best Peak Side Lobe Ratio (PSLR).

For the above codes we have plotted ACF and AF[7,11].

TABLE III. PSLR of Golomb and Welch Costas Sequence for N=11

Order (11)	Length	sequence	PSLR(dB)

Golomb Sequence	72	0 1 4 13 28 33 47 54 64 7072 0 1 9 19 24 31 52 56 58 69 72	-60.1052 -58.6763
Welch Sequence	11	1 3 7 2 5 11 10 8 4 9 6 10 3 4 2 6 11 1 8 7 9 5 6 9 4 8 10 11 5 2 7 3 1 5 9 7 8 1 11 6 2 4 3 10	-24.9044 -22.0351 -21.1328 -20.0557

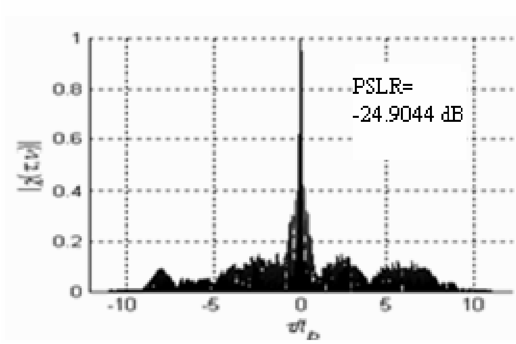

Figure 2.a. Autocorrelation diagram for Welch sequence for N=11[1 3 7 2 5 11 10 8 4 9 6]

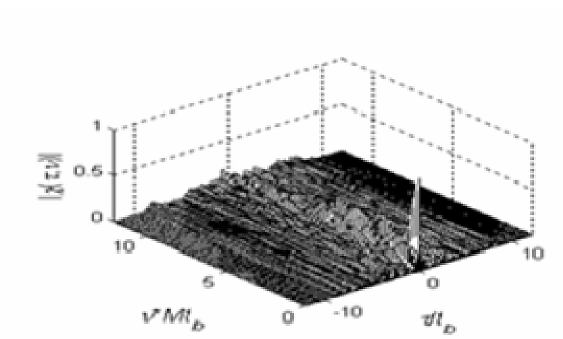

Figure 2.b. Ambiguity function for Welch Costas sequence for N=11[1 3 7 2 5 11 10 8 4 9 6]

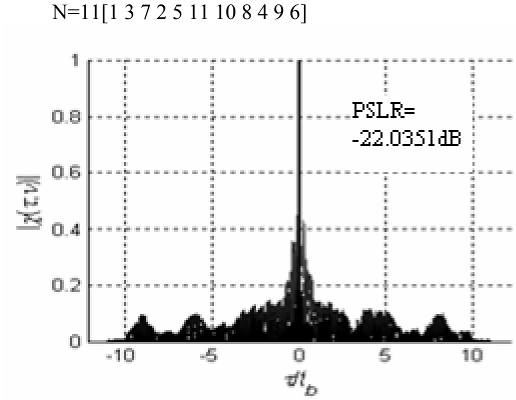

Figure 3.a. Autocorrelation diagram for Welch sequence N=11[10 3 4 2 6 11 1 8 7 9 5]

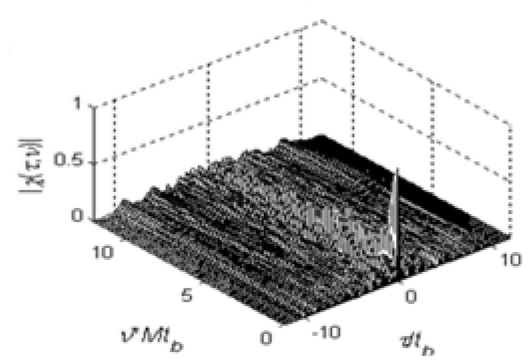

Figure 3.b. Ambiguity function for Welch Costas sequence for N=11[10 3 4 2 6 11 1 8 7 9 5]

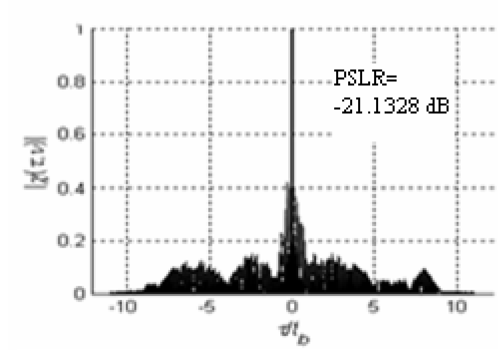

Figure 4.a. Autocorrelation diagram for Welch sequence N=11[6 9 4 8 10 11 5 2 7 3 1]

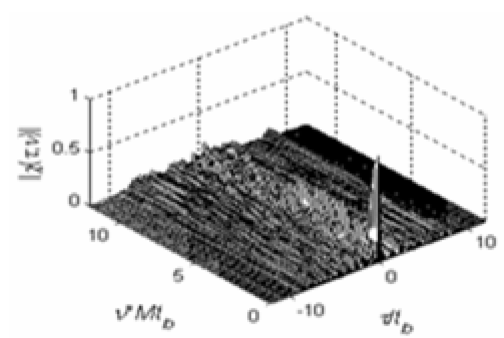

Figure 4.b. Ambiguity function for Welch Costas sequence for N=11[6 9 4 8 10 11 5 2 7 3 1]

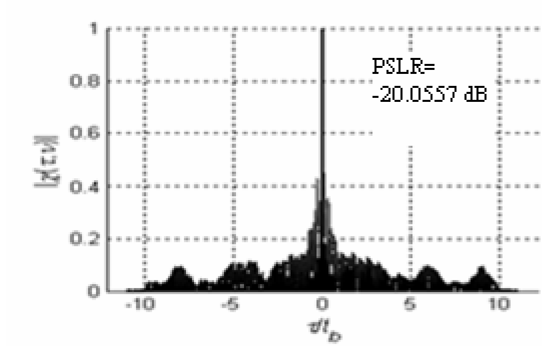

Figure 5.a. Autocorrelation diagram for Welch sequence N=11[5 9 7 8 1 11 6 2 3 10]

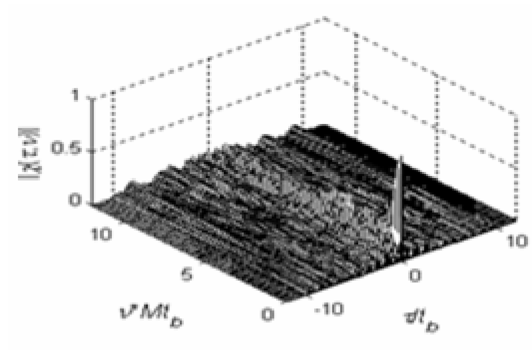

Figure 5.b.
Ambiguity function for Welch Costas sequence for N=11[5 9 7 8 1 11 6 2 3 10]

5.3. FREQUENCY CODED: GOLOMB- SEQUENCE.

For N=11 we have two different Golomb sequence of length 72, which can be used as Costas pulse compression codes as they satisfy the difference triangle formation and as said in section-3, a sequence can be used in Costas pulse compression when they satisfy difference triangle formation. N=11 {0 1 9 19 24 31 52 56 58 69 72, 0 1 9 19 24 31 52 56 58 69 72}. These are the two sequences in which we have taken one code which as best PSLR. For the bove codes we have plotted ACF and AF[6,7,11].

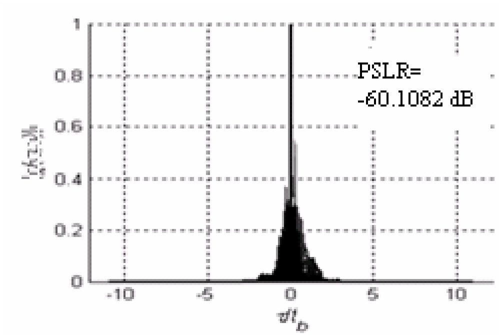

Figure 6.a. Autocorrelation diagram for Golomb sequence N=11[0 1 4 13 28 33 417 54 64 70 72]

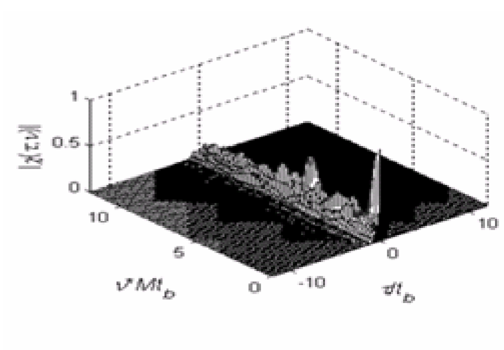

Figure 6.b. Ambiguity function for Golomb sequence for N=11[0 1 4 13 28 33 417 54 64 70 72]

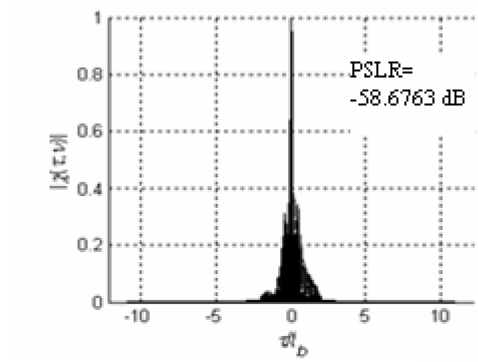

Figure 7.a. Autocorrelation diagram for Golomb sequence N=11[0 9 19 24 31 52 56 58 69 72]

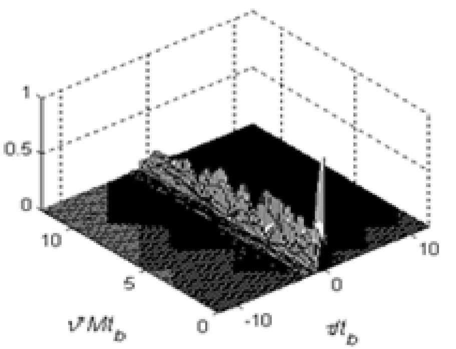

Figure 7.b. Ambiguity function for Golomb sequence for N=11[0 9 19 24 31 52 56 58 69 72]

6. CONCLUSIONS.

We applied Golomb sequence and Welch Costas sequence to the frequency coded Costas sequence with size N=11.

Comparing the ACF of Welch Costas sequence and Golomb Ruler, we noticed that the Golomb Ruler gives getter results with PSLR = -60.1052 dB [0 1 4 13 28 33 47 54 64 70 72]. This is without applying any sidelobe reduction techniques.

In the ambiguity domain the Golomb sequences are exhibiting negligible sidelobes which is visible from the ambiguity diagram with flat sidelobe pattern.

7. ACKNOWLEDGEMENTS

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References

[1] Rihaczek.A.W., *Principles of High Resolution Radar* , McGraw-Hill, New York, 1969.

[2] Costas.J.P., *A study of a class of detection waveforms having nearly ideal range-Doppler ambiguity properties* Proceedings of the IEEE, vol. 72, no. 8, August 1984, pp. 996-1009.

[3] Levanon.N., Mozeson, *Stepped frequency pulse train radar signal*

,IEEE Proc. Sonar Navig., 2002, 149, (6) pp. 297-309.

[4] Golomb.S.W., and H.Taylor *Constructions and properties of Costas arrays*, Proceedings of the IEEE, vol. 72 no. 9, September 1984, pp. 1143- 1163.

[5] Levanon.N., Mozeson *Nullifying ACF grating lobe stepped-frequency train of LFM pulses* IEEE Transaction on Aerospece and Electronics Systems, Vol39, (2) (Apr.2003), pp. 694-703.

[6] Levanon. N, Mozeson *Radar Signal,* 2004.

[7] Levanon.N, *Radar Prirzciples* Wiley, New York, 1988.

[8] Apostolos Dimitromanolakis, *Analysis of the Golomb Ruler and theSidon Set Problems, and Determination of Large, Near-Optimal Golomb Rulers*Technical University of Crete, June 2002.

[9] K. Kasas, H. Aboulnour, O. Kawas *Improving Ambiguity Function of CostasSigna* Mediterranean Microwave Symposium, Nov. 15-17, 2009.

[10] K. Kasas, H. Aboulnour, O. Kawas, Improving Costas Radar Signal by Applying Variable Time Spacing Using Costas Array And Golomb Ruler, Damascus University Journal Vol. (26) - No. (2) 2010 .

[11] Bassem R. Mahafza Radar systems analysis and design using Matlab, Chapman & Hall/CRC, 2000.

[12] M.J.E.Golay., Sieves for low auto correlation binary sequences, IEEE Transactions on information theory, vol 23,43-51,1977.

AUTHORS PROFILE

A. NagaJyothi

K. Raja Rajeswari

A.NagaJyothi was born in 1982 at Visakhapatnam. She received her B.Tech (ECE) from Nagarjuna University and M.Tech(Radar & Microwave Engneering) from AndhraUniversity College of Engineering(A). She has a teaching experience of 3 years. Presently she is perusing her Ph.D in the area of Signal Processing in Andhra University, Visakhapatnam.

K. Raja Rajeswari obtained her B.E., M.E. and Ph.D. degrees from Andhra University, Visakhapatnam, India in 1976, 1978 and 1992 respectively. Presently she is professor in the Department of Electronics and Communication Engineering, Andhra University. She is Dean for Quality Assurance, Andhra University College of Engineering. She has published over 100 papers in various National, International Journals and conferences. She is author of the textbook *Signals and Systems* published by PHI. She is co-author of the textbook *Electronics Devices and Circuits* published by Pearson Education. Her research interests include Radar and Sonar Signal Processing, Wireless Communication Technologies. She has guided *twelve* Ph.D.s and presently she is guiding *fifteen* students for Doctoral degree. She is immediate past chairperson of IETE, Visakhapatnam Centre. Present she is Governing Council Member of IETE, New Delhi. She is recipient of prestigious IETE Prof SVC Aiya Memorial National Award for the year 2009, Best Researcher Award by Andhra University for the year 2004 and Dr. Sarvepalli Radhakrishnan Best Academician Award of the year 2009 by Andhra University. She is expert member for various national level academic and research committees and reviewer for various national/international journals.