Array Processing for Multi-User Systems with Feedback

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Abstract— In this paper, we investigate how to use array processing to achieve full diversity and low decoding complexity for multi-user system. First, we assume that we have 2 transmitters and 2 receivers. Different transmitter sends code words to different receiver at the same time. Due to the existence of interference, how to tackle this problem is not straightforward.
We propose an orthogonal transmission scheme that combines space-time codes and array processing to achieve low-complexity decoding and full diversity for transmitted signals. Simulation results validate our theoretical analysis.

Key Words— Array processing, full diversity, coding design, interference cancellation, feedback.

I. Introduction

The prospect of extraordinary improvements in the capacity of wireless networks has drawn considerable attention to multiple-input multiple-output (MIMO) communication techniques. MIMO methods make use of multi-element antenna arrays at both the transmit and the receive side of a radio link to drastically improve the capacity over more traditional single-input multiple-output (SIMO) systems (with multiple antennas typically being used at the base station only). SIMO channels can provide diversity gain, array gain, and interference canceling gain. In addition to these advantages, MIMO links can offer a so-called multiplexing gain by opening parallel spatial data pipes or channels within the same frequency band at no additional power expenditure. In the presence of rich multipath leading to antenna decorrelation and full channel rank, MIMO links offer capacity gains that are proportional to the minimum of the number of transmit and receive antennas. These gains can be achieved using spatial multiplexing algorithms (a.k.a. BLAST). In the presence of channel rank loss one resorts to more robust lower rate transmission techniques based on space time codes [1]–[6].

Recently, several space-time processing techniques have been used in multiple access channels to reduce the decoding complexity and enhance system performance by canceling the interference from different users [7]–[31].

In this paper, inspired by the pioneer study of wireless multi-user network with space-time processing in [5], we investigate how to achieve the low complexity decoding and the highest possible diversity to improve the transmission quality for space-time codes in Z channels without losing symbol rate. Our idea to solve this problem is to design proper precoding and decoding schemes based on space-time coding with the assumption of full channel information at the transmitter. The idea of combining space-time coding and precoding in multi-user systems is not new [12], [13]. Note that one can use interference alignment methods to achieve the highest degree of freedom [16]. But under our assumptions, using interference alignment, the diversity will be one. The concentration of this paper is to achieve the highest diversity with low decoding complexity for space-time codes, not achieving the highest degree of freedom.

The outline of the paper follows next. Section II introduces our motivation and the Z channels we discuss in this paper. In Section III, we propose an orthogonal transmission scheme which is necessary to achieve low complexity decoding, high coding gain and full diversity as shown in later sections. In Section IV, our decoding scheme is proposed. We analyze the performance of our scheme in Section V. Simulation results are presented in Section VI and Section VII concludes the paper.

Notation: We use boldface letters to denote matrices and vectors, super-scripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^\dagger$ to denote transpose, conjugate and transpose conjugate, respectively. We denote the element in the ith row and the jth column of matrix $\mathbf{X}$ by $X(i, j)$. We denote the jth column of a matrix $\mathbf{X}$ by $\mathbf{X}(j)$.

II. Motivation and Channel Model

We assume there are 2 users each with 2 transmit antennas and 2 receivers each with 2 receive antennas. Both users want to send different space-time codes to Receivers 1 and 2 on the same frequency band at the same time. As shown in Figure 1, User 1 wants to send code word C to Receiver 1 without causing interference to Receiver 2. User 2 wants to send code word S to Receiver 2 and causes interference to Receiver 1. When channel knowledge is not available at the transmitters, space-time codes combined with TDMA can be used to achieve symbol by symbol decoding and full diversity. But the symbol rate reduces to one half. A solution to keep the symbol rate unchanged when space-time codes are used, is to combine space-time coding and array processing. In other words, we allow all transmitters to send space-time codes simultaneously to keep rate one and utilize special array processing techniques to achieve low complexity decoding and full diversity. In this paper, we achieve the above goals under short-term power constraints, fixed code word block length and limited delay, when there is outage. We do not claim that our scheme can achieve capacity or full degree of freedom. After all, there is a tradeoff between diversity and multiplexing gain, which is outside the scope of this paper.

First, let us introduce the input-output equations. We let

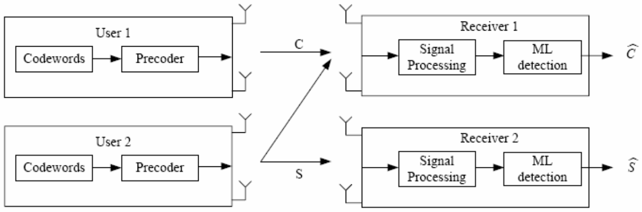

Fig. 1. Z Channel

each user transmit Alamouti Codes [17] as follows:

$$\mathbf{C} = \begin{pmatrix} c_1 & -c_2^* \\ c_2 & c_1^* \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix} \tag{1}$$

where $i, j = 1, 2$. Note that we can also use other space-time codes with rate one and Alamouti code is just one example. Let

$$\mathbf{A}^t = \mathbf{A}^t(2,2) \tag{2}$$

be the precoders we need to design. In this paper, we use (i, j) denote a matrix of $i \times j$ dimension. They are combined with the space-time codes sent by User 1 and this is the first step of our array processing technique. Note that in order to satisfy the short-term power constraint, we need

$$||\mathbf{A}^t(2,2)||_F^2 = 1 \tag{3}$$

Similarly, the precoders for User 2 is defined as

$$\mathbf{B}^t = \mathbf{B}^t(2,2) \tag{4}$$

with the power constraint

$$||\mathbf{B}^t(2,2)||_F^2 = 1 \tag{5}$$

The channels are quasi-static flat Rayleigh fading and keep unchanged during two time slots. Then we let

$$\mathbf{H}_l = \mathbf{H}_l(2,2) \tag{6}$$

denote the channel matrix between User 1 and Receivers l, respectively. Similarly, we use

$$\mathbf{G}_l = \mathbf{G}_l(2,2) \tag{7}$$

to denote the channel matrix between User 2 and Receiver l, respectively. Then the received signals at Receiver 1 at time slot t can be denoted by

$$\mathbf{y}_1^t(2,1) = \mathbf{H}_1(2,2)\mathbf{A}^t(2,2)\mathbf{C}(t) + \mathbf{G}_1(2,2)\mathbf{B}^t(2,2)\mathbf{S}(t) + \mathbf{n}_1^t(2,1) \tag{8}$$

where

$$\mathbf{y}_1^t = \mathbf{y}_1^t(2,1), \quad \mathbf{n}_1^t = \mathbf{n}_1^t(2,1) \tag{9}$$

denote the received signals and the noise at Receiver 1, respectively, at time slot t. Similarly, at time slot t, Receiver 2 will receive the following signals

$$\mathbf{y}_2^t(2,1) = \mathbf{G}_2(2,2)\mathbf{B}^t(2,2)\mathbf{S}(t) + \mathbf{n}_2^t(2,1) \tag{10}$$

where

$$\mathbf{y}_2^t = \mathbf{y}_2^t(2,1), \quad \mathbf{n}_2^t = \mathbf{n}_2^t(2,1) \tag{11}$$

Equations (8) and (10) are the channel equations on which we will base our design in this paper.

III. Precoder Design and Orthogonal Transmission Structure

In this section, we will build an orthogonal transmission structure by combining the space-time codes and our precoders. This orthogonal transmission structure is necessary because it provides two benefits. The first benefit is that low-complexity decoding can be realized because under this orthogonal transmission structure, different codewords will be sent along different orthogonal vectors. We can easily decode the symbols without the interference at each receiver. The second benefit is that we can achieve full diversity and higher coding gain once we make the proper array processing as shown in later sections. This is the key difference between our array processing method and the interference alignment method. The latter can only achieve the first benefit. Of course, the tradeoff is that we lose the maximum possible degree of freedom in the process.

Different users and different codewords may have different diversities. By saying full diversity, we mean the diversity is full for each codeword sent by each user. For example, full diversity for User 1 means at Receiver 1, the diversity for codeword **C** is full. Similarly, by saying the diversity for User 2 is full, we mean that at Receiver 2, the diversity for codeword **S** is full. In this section, we show how to build the orthogonal transmission structure by designing proper precoders. Later, we will show that our proposed orthogonal transmission scheme can achieve low-complexity decoding and full diversity.

Our main idea to build the orthogonal transmission structure is to adjust each signal in the signal space of Z channels by using precoders for each transmitter, such that at the receiver each desired signal is orthogonal to all other signals. In Equation (8), we use

$$\mathbf{H}_{11}^t(2,2) = \mathbf{H}_1(2,2)\mathbf{A}^t(2,2), \mathbf{G}_{11}^t(2,2) = \mathbf{G}_1(2,2)\mathbf{B}^t(2,2) \tag{12}$$

to denote the equivalent channel matrices. Then Equation (8) becomes

$$\mathbf{y}_1^t(2,1) = \mathbf{H}_{11}^t(2,2)\mathbf{C}(t) + \mathbf{G}_{11}^t(2,2)\mathbf{S}(t) + \mathbf{n}_1^t(2,1) \tag{13}$$

Similarly, in Equation (10), if we use

$$\mathbf{G}_{21}^t(2,2) = \mathbf{G}_2(2,2)\mathbf{B}^t(2,2) \tag{14}$$

to denote the equivalent channel matrices, we have

$$\mathbf{y}_2^t(2,1) = \mathbf{G}_{21}^t(2,2)\mathbf{S}(t) + \mathbf{n}_2^t(2,1) \tag{15}$$

By Equation (13), since the receiver has 2 receive antennas, each symbol is actually transmitted along a 2-dimensional vector in a 2-dimensional space. Because each user sends two symbols at the same time, at the receiver, there are 4 signal vectors in the two-dimensional space.

Since we want to send $\mathbf{C}$ and $\mathbf{S}$ along orthogonal directions, we let each one of $\mathbf{C}, \mathbf{S}$ occupy only one dimension. In other words, for any codeword, we should transmit each of the corresponding four symbols in the same direction. In this way, there are only 2 transmit directions. Once we can align the 2 transmit directions of $\mathbf{C}$, $\mathbf{S}$ properly, we can separate them completely. This is the main idea to build the orthogonal transmission structure. Note that this is only a general idea and much details are omitted. For example, we will show later that after some array processing and moving the interference at the receiver, each symbol at each receiver will have its own direction. We need to do additional array processing to reduce the decoding complexity and achieve full diversity.

In this section, we only explain the above main idea. By Equation (13), c_1, c_2 are transmitted along $\mathbf{H}_{11}^t(2,2)(1), \mathbf{H}_{11}^t(2,2)(2)$, respectively. In order to make $\mathbf{H}_{11}^t(2,2)(1), \mathbf{H}_{11}^t(2,2)(2)$ along the same direction, by Equation (12), we need

$$\mathbf{A}^t(2,2)(1) = \frac{1}{\alpha^t}\mathbf{A}^t(2,2)(2) \tag{16}$$

where α_{11}^t is a constant that we will determine later. From $||\mathbf{A}^t(2,2)||_F^2 = 1$, we know

$$||\mathbf{A}^t(2,2)(1)||_F^2 = \frac{1}{1+(\alpha^t)^2} \tag{17}$$

So when we design precoder $\mathbf{A}^t(2,2)$, Equations (16) and (17) should be satisfied. Similarly, precoders $\mathbf{B}^t(2,2)$ should also satisfy the following conditions:

$$\mathbf{B}^t(2,2)(1) = \frac{1}{\beta^t}\mathbf{B}_1^t(2,2)(2) \tag{18}$$

with

$$||\mathbf{B}^t(2,2)(1)||_F^2 = \frac{1}{1+(\beta^t)^2} \tag{19}$$

Now Equations (13) and (15) become

$$\mathbf{y}_1^t(2,1) = [\mathbf{H}_{11}^t(2,2)(1), \mathbf{H}_{11}^t(2,2)(1)] \cdot \mathbf{C}(t) + [\mathbf{G}_{11}^t(2,2)(1), \mathbf{G}_{11}^t(2,2)(1)] \cdot \mathbf{S}(t) + \mathbf{n}_1^t(2,1) \tag{20}$$

and

$$\mathbf{y}_2^t(2,1) = [\mathbf{G}_{21}^t(2,2)(1), \mathbf{G}_{21}^t(2,2)(1)] \cdot \mathbf{S}(t) + \mathbf{n}_2^t(2,1) \tag{21}$$

where $\mathbf{H}_{11}^t(2,2)(1), \mathbf{G}_{11}^t(2,2)(1), \mathbf{G}_{21}^t(2,2)(1)$ denote the first column of matrix $\mathbf{H}_{11}^t(2,2)$, $\mathbf{G}_{11}^t(2,2)$, $\mathbf{G}_{21}^t(2,2)$, respectively. At receiver one, after we combine the channel equations in two time slots, we have

$$\mathbf{y}_1(2,1) = \begin{pmatrix} \mathbf{H}_{11}^1(2,2)(1) & \mathbf{H}_{11}^1(2,2)(1) \\ (\mathbf{H}_{11}^2(2,2)(1))^* & -(\mathbf{H}_{11}^2(2,2)(1))^* \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \mathbf{G}_{11}^1(2,2)(1) & \mathbf{G}_{11}^1(2,2)(1) \\ (\mathbf{G}_{11}^2(2,2)(1))^* & -(\mathbf{G}_{11}^2(2,2)(1))^* \end{pmatrix} \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \mathbf{n}_1(2,1) \tag{22}$$

where

$$\mathbf{y}_1(2,1) = \begin{pmatrix} \mathbf{y}_1^1(2,1) \\ (\mathbf{y}_1^2(2,1))^* \end{pmatrix} \tag{23}$$

and

$$\mathbf{n}_1(2,1) = \begin{pmatrix} \mathbf{n}_1^1(2,1) \\ (\mathbf{n}_1^2(2,1))^* \end{pmatrix} \tag{24}$$

Similarly, at receiver two, after we combine the channel equations in two time slots, we have

$$\mathbf{y}_2(2,1) = \begin{pmatrix} \mathbf{G}_{21}^1(2,2)(1) & \mathbf{G}_{21}^1(2,2)(1) \\ (\mathbf{G}_{21}^2(2,2)(1))^* & -(\mathbf{G}_{21}^2(2,2)(1))^* \end{pmatrix} \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \mathbf{n}_2(2,1) \tag{25}$$

where

$$\mathbf{y}_2(2,1) = \begin{pmatrix} \mathbf{y}_2^1(2,1) \\ (\mathbf{y}_2^2(2,1))^* \end{pmatrix} \tag{26}$$

and

$$\mathbf{n}_2(2,1) = \begin{pmatrix} \mathbf{n}_2^1(2,1) \\ (\mathbf{n}_2^2(2,1))^* \end{pmatrix} \tag{27}$$

By Equation (22), we can see that once we make vector $\mathbf{H}_{11}^1(2,2)(1)$ orthogonal to $\mathbf{G}_{11}^1(2,2)(1)$ at time slot 1 and $\mathbf{H}_{11}^2(2,2)(1)$ orthogonal to $\mathbf{G}_{11}^2(2,2)(1)$ at time slot 2, signal vectors for c_1, c_2 will lie in a subspace which is orthogonal to the subspace created by the signal vectors for s_1, s_2. Because of this orthogonality, at the receiver one, we can easily separate the desired signals c_1, c_2 from the interference signals s_1, s_2. At Receiver 2, since there is no interference, by Equation (25), we can easily decode the desired signals s_1, s_2. This is our main idea to achieve interference-free transmission in this interference channel.

Now we show how to derive the above orthogonality by designing precoders for Users 1 and 2 simultaneously. Assume the Singular Value Decomposition of channel matrices $\mathbf{H}_1(2,2)$, $\mathbf{G}_2(2,2)$ as follows

$$\mathbf{H}_1(2,2) = \mathbf{V}_{H_1(2,2)}\mathbf{\Lambda}_{H_1(2,2)}\mathbf{U}_{H_1(2,2)}^\dagger \tag{28}$$

$$\mathbf{G}_2(2,2) = \mathbf{V}_{G_2(2,2)}\mathbf{\Lambda}_{G_2(2,2)}\mathbf{U}_{G_2(2,2)}^\dagger \tag{29}$$

At time slot 1, we let User 1 transmit along its best direction. In this case, we can choose the precoder

$$\mathbf{A}^1(2,2) = \frac{1}{\sqrt{2}}[\mathbf{U}_{H_1(2,2)}(2,2)(1), \mathbf{U}_{H_1(2,2)}(2,2)(1)] \tag{30}$$

Then we design precoders for User 2 such that at both receivers 1, the signal vectors from User 2 are orthogonal to the signal vectors of User 1. Note that at receiver one, the signal from User 2 is interference and at receiver two, the signal from User 2 is the desired signal. We need to consider the signals at both receivers when we design precoder for User 2. So, at time slot 1, the precoder $\mathbf{B}^1$ needs to satisfy the following three equations. At receiver one ,we need

$$(\mathbf{H}_{11}^1(2,2)(1))^\dagger \cdot \mathbf{G}_{11}^1(2,2)(1) = 0 \tag{31}$$

and the power constraint

$$||\mathbf{B}^1(2,2)(1)||^2 = \frac{1}{2} \tag{32}$$

By solving the above two equations, we can get the precoder $\mathbf{B}^1(2,2)$ for User 2 at time slot 1. At time slot 2, we let User 2 transmit along its best direction. In this case, we can choose the precoder

$$\mathbf{B}^2(2,2) = \frac{1}{\sqrt{2}}[\mathbf{U}_{G_2(2,2)}(2,2)(1), \mathbf{U}_{G_2(2,2)}(2,2)(1)] \tag{33}$$

For User 1 at time slot 2, at receiver one, $\mathbf{A}^2(2,2)$ needs to satisfy the following the following equation

$$(\mathbf{G}_{11}^2(2,2)(1))^\dagger \cdot \mathbf{H}_{11}^2(2,2)(1) = 0 \tag{34}$$

and the power constraint

$$||\mathbf{A}^2(2,2)(1)||^2 = \frac{1}{2} \tag{35}$$

By solving the above two equations, we can get the precoder $\mathbf{A}^2(2,2)$ for User 1 at time slot 2. With our precoders $\mathbf{A}^1(2,2)$, $\mathbf{B}^1(2,2)$ at time slot 1 and $\mathbf{A}^2(2,2)$, $\mathbf{B}^2(2,2)$ at time slot 2, we can show that we can achieve interference-free transmission with low decoding complexity and full diversity simultaneously as shown in the next two sections.

IV. Decoding with Low Complexity

In the last section, we have shown how to build the orthogonal transmission structure. Once the orthogonal structure is built, it is easy to realize low-complexity decoding. In this section, we will show how to decode and analyze the decoding complexity. We first consider the decoding at receiver one. In Equation (22), if we let

$$\overline{\mathbf{H}}_1(2,2) = \begin{pmatrix} \mathbf{H}_{11}^1(2,2)(1) & \mathbf{H}_{11}^1(2,2)(1) \\ (\mathbf{H}_{11}^2(2,2)(1))^* & -(\mathbf{H}_{11}^2(2,2)(1))^* \end{pmatrix} \tag{36}$$

and

$$\overline{\mathbf{G}}_1(2,2) = \begin{pmatrix} \mathbf{G}_{11}^1(2,2)(1) & \mathbf{G}_{11}^1(2,2)(1) \\ (\mathbf{G}_{11}^2(2,2)(1))^* & -(\mathbf{G}_{11}^2(2,2)(1))^* \end{pmatrix} \tag{37}$$

then Equation (22) becomes

$$\mathbf{y}_1(2,1) = \overline{\mathbf{H}}_1(2,2) \cdot \binom{c_1}{c_2} + \overline{\mathbf{G}}_1(2,2) \cdot \binom{s_1}{s_2} + \mathbf{n}_1(2,1) \tag{38}$$

Note that at receiver one, $\binom{c_1}{c_2}$ are the desired signal and $\binom{s_1}{s_2}$ are the interference. We can cancel the interference by multiplying both sides of Equation (38) by matrix $\overline{\mathbf{H}}_1(2,2)^\dagger$. Then we get

$$\overline{\mathbf{H}}_1(2,2)^\dagger \cdot \mathbf{y}_1(2,1) = \overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2) \binom{c_1}{c_2} + \overline{\mathbf{H}}_1(2,2)^\dagger \mathbf{n}_1(2,1) \tag{39}$$

Here we have canceled the interference because $\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{G}}_1(2,2) = 0$. In order to decode the symbols, we first multiply both sides of Equations (39) by matrix $(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{-\frac{1}{2}}$ to whiten the noise, i.e.,

$$\begin{aligned} &(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{-\frac{1}{2}} \overline{\mathbf{H}}_1(2,2)^\dagger \cdot \mathbf{y}_1(2,1) = \\ &(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{\frac{1}{2}} \binom{c_1}{c_2} + \\ &(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{-\frac{1}{2}} \overline{\mathbf{H}}_1(2,2)^\dagger \mathbf{n}_1(2,1) \end{aligned} \tag{40}$$

Then we can detect (c_1, c_2) by

$$\begin{aligned} &\widehat{c}_1, \widehat{c}_2 = \\ &\arg\min_{c_1,c_2} \Big\| (\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{-\frac{1}{2}} \overline{\mathbf{H}}_1(2,2)^\dagger \\ &\times \mathbf{y}_1(2,1) - (\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{\frac{1}{2}} \binom{c_1}{c_2} \Big\|_F^2 \end{aligned} \tag{41}$$

Further, note that

$$\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2) = \begin{pmatrix} ||\mathbf{H}_{11}^1(2,2)(1)||^2 + ||\mathbf{H}_{11}^2(2,2)(1)||^2 & ||\mathbf{H}_{11}^1(2,2)(1)||^2 - ||\mathbf{H}_{11}^2(2,2)(1)||^2 \\ ||\mathbf{H}_{11}^1(2,2)(1)||^2 - ||\mathbf{H}_{11}^2(2,2)(1)||^2 & ||\mathbf{H}_{11}^1(2,2)(1)||^2 + ||\mathbf{H}_{11}^2(2,2)(1)||^2 \end{pmatrix} \tag{42}$$

So, when QAM is used, Equation (40) is equivalent to the following two equations.

$$\begin{aligned} &\mathrm{Real}\{(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{-\frac{1}{2}} \overline{\mathbf{H}}_1(2,2)^\dagger \cdot \mathbf{y}_1(2,1)\} \\ &= (\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{\frac{1}{2}} \mathrm{Real}\{\tbinom{c_1}{c_2}\} \\ &+ \mathrm{Real}\{(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{-\frac{1}{2}} \overline{\mathbf{H}}_1(2,2)^\dagger \mathbf{n}_1(2,1)\} \end{aligned} \tag{43}$$

$$\begin{aligned} &\mathrm{Imag}\{(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{-\frac{1}{2}} \overline{\mathbf{H}}_1(2,2)^\dagger \cdot \mathbf{y}_1(2,1)\} \\ &= (\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{\frac{1}{2}} \mathrm{Imag}\{\tbinom{c_1}{c_2}\} \\ &+ \mathrm{Imag}\{(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{-\frac{1}{2}} \overline{\mathbf{H}}_1(2,2)^\dagger \mathbf{n}_1(2,1)\} \end{aligned} \tag{44}$$

So we can detect the real part and the imaginary part of c_1, c_2 separately as follows:

$$\mathrm{Real}\{\widehat{c}_1, \widehat{c}_2\} = \arg \min_{\mathrm{Real}\{c_1,c_2\}} \Big\| \mathrm{Real}\{(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{-\frac{1}{2}} \overline{\mathbf{H}}_1(2,2)^\dagger \cdot \mathbf{y}_1(2,1)\} - (\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{\frac{1}{2}} \mathrm{Real}\{\left(\begin{smallmatrix} c_1 \\ c_2 \end{smallmatrix}\right)\} \Big\|_F^2 \quad (45)$$

$$\mathrm{Imag}\{\widehat{c}_1, \widehat{c}_2\} = \arg \min_{\mathrm{Imag}\{c_1,c_2\}} \Big\| \mathrm{Imag}\{(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{-\frac{1}{2}} \overline{\mathbf{H}}_1(2,2)^\dagger \cdot \mathbf{y}_1(2,1)\} - (\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{\frac{1}{2}} \mathrm{Imag}\{\left(\begin{smallmatrix} c_1 \\ c_2 \end{smallmatrix}\right)\} \Big\|_F^2 \quad (46)$$

The decoding complexity is symbol-by-symbol. Similarly, we can detect s_1, s_2 with symbol-by-symbol complexity at receiver two.

V. Diversity Analysis

Analyzing the diversity of multiuser MIMO system is very challenging due to the existence of multi-antenna and multi-path interference. This problem is solved successfully by [5] where the authors proposed an original way for the diversity analysis using matrix theory. This method is very powerful because it can be used in any multi-antenna interference environment. Here we use their proposed methods to do the diversity analysis. First, the diversity is defined as

$$d = -\lim_{\rho \to \infty} \frac{\log P_e}{\log \rho} \quad (47)$$

where ρ denotes the SNR and P_e represents the probability of error. We let $\mathbf{e} = \left(\begin{smallmatrix} e_1 \\ e_2 \end{smallmatrix}\right) = \left(\begin{smallmatrix} c_1 \\ c_2 \end{smallmatrix}\right) - \begin{pmatrix} \widehat{c}_1 \\ \widehat{c}_2 \end{pmatrix}$ denote the error vector. Here we add a rotation matrix $\mathbf{R}$ on the transmitted codewords to improve the system performance. Based on Equation (40), the pairwise error probability (PEP) for c_1, c_2 can be written as [18]

$$\begin{aligned} &P(\mathbf{c} \to \overline{\mathbf{c}} | \overline{\mathbf{H}}_1(2,2)) = \\ &Q\left(\sqrt{\frac{\rho \|(\overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2))^{\frac{1}{2}} \mathbf{R}(2,2)\mathbf{e}(2,1)\|_F^2}{4}}\right) = \\ &Q\left(\sqrt{\frac{\mathbf{e}(2,1)^\dagger \mathbf{R}(2,2)^\dagger \overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2)\mathbf{R}(2,2)\mathbf{e}(2,1)}{4/\rho}}\right) \\ &\le \exp(\\ &\frac{\mathbf{e}(2,1)^\dagger \mathbf{R}(2,2)^\dagger \overline{\mathbf{H}}_1(2,2)^\dagger \overline{\mathbf{H}}_1(2,2)\mathbf{R}(2,2)\mathbf{e}(2,1)}{-4}) \\ &= \exp\left(-\frac{\rho\lambda}{4/\rho}\right) \end{aligned} \quad (48)$$

where

$$\lambda = \|\mathbf{H}_{11}^1(2,2)(1)\|_F^2 |\widehat{e}_1 + \widehat{e}_2|^2 + \|\mathbf{H}_{11}^2(2,2)(1)\|_F^2 |\widehat{e}_1 - \widehat{e}_2|^2 \quad (49)$$

and

$$\widehat{\mathbf{e}}(2,1) = \begin{pmatrix} \widehat{e}_1 \\ \widehat{e}_2 \end{pmatrix} = \mathbf{R}(2,2)\mathbf{e}(2,1) \quad (50)$$

Since

$$\|\mathbf{H}_{11}^1(2,2)(1)\|_F^2 \ge \frac{\|\mathbf{H}_1(2,2)\|_F^2}{2} \cdot \frac{1}{2} \quad (51)$$

Inequality (48) can be written as

$$\begin{aligned} P(\mathbf{c} \to \overline{\mathbf{c}} | \overline{\mathbf{H}}_1(2,2)) &\le \exp\left(-\frac{\rho\lambda}{4}\right) \\ &= \exp\left(-\frac{\rho \|\mathbf{H}_1(2,2)\|_F^2 |\widehat{e}_1 + \widehat{e}_2|^2}{16}\right) \end{aligned} \quad (52)$$

Therefore, we have

$$\begin{aligned} P(\mathbf{c} \to \overline{\mathbf{c}}) &= E[P(\mathbf{c} \to \overline{\mathbf{c}} | \overline{\mathbf{H}}_1(2,2))] \\ &= E\left[\exp\left(-\frac{\rho \|\mathbf{H}_1(2,2)\|_F^2 |\widehat{e}_1 + \widehat{e}_2|^2}{16}\right)\right] \\ &= \frac{1}{\prod_{j=1}^4 (1 + \frac{\rho\tau}{16})} \end{aligned} \quad (53)$$

where

$$\tau = |\widehat{e}_1 + \widehat{e}_2|^2 \quad (54)$$

At high SNR region, (53) can be written as

$$P(\mathbf{c} \to \overline{\mathbf{c}}) \le \left(\frac{\rho\tau}{16}\right)^{-4} \quad (55)$$

So the diversity is 4, full diversity, as long as $\tau \neq 0$. Also the coding gain is affected by τ and we can choose rotation matrix $\mathbf{R}(2,2)$ properly to maximize τ. The best choice for rotation matrix depends on the adopted constellation. Such an optimization is a straightforward optimization that has been discussed in many existing literature [19]. Similarly, we can prove that the diversity for other codewords is also full.

VI. Simulation Results

In this section, we provide simulation results to evaluate the performance of the proposed scheme. First, we assume there are 2 transmitters each with 2 transmit antennas and 2 receivers each with 2 antennas. Each user uses our proposed scheme to transmit Alamouti codes to its receiver. Figure 2 presents simulation results using 8-PSK. We compare the performance of our scheme with that of two other scenarios that can achieve interference cancellation. In the first scenario, we use TDMA and beamforming. That is, at each time slot, only one transmitter sends signals to one receiver using beamforming. 64-QAM is used to have the same bit-rate. In the second scenario, each user uses the well-known multi-user interference cancellation technique with beamforming scheme proposed in [5]. The results show that our proposed scheme can achieve full diversity and symbol rate one. Note that we combine the array processing and space-time coding to avoid symbol rate loss. This does not mean that we cannot change the bit rate. We can always adapt the bit rate by changing the constellation according to the channel condition. In comparison, the TDMA and beamforming method can achieve full diversity but the rate is one half. The MUD method can achieve full rate, but it requires feedback. As shown in the figure, our scheme provides the best performance due to its high diversity and increased coding gain without any rate loss.

VII. CONCLUSIONS

In this paper, we propose a transmission scheme for Z channels with two transmitters each with two transmit antennas and two receivers each with two receive antennas. We achieve full diversity and low decoding complexity, by combining array processing and space-time coding. We analytically prove that our scheme can achieve low-complexity decoding and full diversity. Simulation results validate our theoretical analysis.

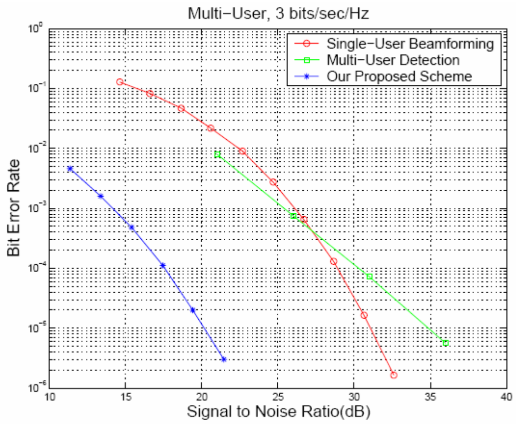

Fig. 2. Simulation results for 2 users each with 2 transmit antennas and 2 receivers each with 2 receive antennas. The constellation is 8-PSK.

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