Different Types of Matrices in Fuzzy Soft Set Theory and Their Application in Decision Making Problems

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Abstract: The purpose of this paper is to define different types of matrices in fuzzy soft set theory. We have introduced here some new operations on these matrices and discussed here all these definitions and operations by appropriate examples. Moreover a new efficient solution procedure has been developed to solve fuzzy soft set based real life decision making problems which may contain more than one decision maker.

Keywords: *Fuzzy Soft Set, Fuzzy Soft Matrix, Choice Matrix, Application.*

I Introduction

For handling real life ambiguous situations we need methodologies which provide some form or other flexible information processing capacity. Soft set theory [6] and [7] is generally used to solve such problems. Initially Molodtsov [3] presented soft set as a completely generic mathematical instrument for modeling uncertainties in the year 1999. Since there is hardly any limitation in describing the objects, researchers simplify the decision making process by selecting the form of parameters they require and subsequently makes it more efficient in the absence of partial information. Maji et al. have done further research on soft set theory [6] and [7] and on fuzzy soft set theory [8]. Moreover Maji et al. [6] utilize the thinking of attributes reduction in rough set theory to reduce parameters set of a soft set. But unreal optimal choice object may be also obtained through this way. So Chen et al.[1] showed that the method of attributes reduction in rough set theory can not simply transplant to parameters reduction in soft set theory, but the elaborate process of parameters reduction in soft set theory is not described by him. Zou et al [10] have given the detailed process of parameter reduction by help of SQL (Structured Query Language) introduced by Chambelin and Boyce [2]. Moreover Zhi Kong et al.[12] have given a new definition of normal parameter reduction of fuzzy soft sets and proposed an algorithm for it. Furthermore, Zou and Xiao[11]presented data analysis approaches of soft sets under incomplete information. These approaches presented in [11] are preferable for reflecting actual states of incomplete data in soft sets. But the applications of soft set theory generally solve problems with the help of rough sets or fuzzy soft sets. Cagman et al [4] and [5] introduced a new soft set based decision making method (uni-int decision making method) [5] which selects a set of optimum elements from the alternatives. They also gave the definition of soft matrices which are representations of soft sets. This representation has several advantages. It is easy to store and manipulate matrices and hence the soft sets represented by them in a computer. Cagman et al. [4] have also proposed an algorithm for solving soft set based decision making problems using several operations of soft matrices defined by them. But for the soft set based decision making problems involving more than two decision makers the method introduced by Cagman is so lengthy and it's computational complexity is also very high. So in [9] first we have defined different types of soft matrices, choice matrix and introduced some operations on them and then proposed a new algorithm using these choice matrices and newly proposed operations of soft matrices to solve soft set based decision making problems. The speciality of this new approach is that it may solve any soft set based decision making problem involving huge number of decision makers very easily and the computational procedure is also less complicated.

In this presentation we have proposed the concept of fuzzy soft matrix. Then we have defined different types of fuzzy soft matrix with giving proper examples. Here we have also proposed the concept of choice matrix associated with a fuzzy soft set. Moreover we have introduced some operations on fuzzy soft matrices and choice matrices. At last we have presented a new algorithm using these choice matrices and newly proposed operations of fuzzy soft matrices to solve fuzzy soft set based decision making problems. The speciality of this new approach is that it may solve any fuzzy soft set based decision making problem involving huge number of decision makers very easily and the computational procedure is so simple.

II Preliminaries

A Definition: [7] Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Let $A \subseteq E$. A pair (F_A, E) is called a **soft set** over U, where F_A is a mapping given by, $F_A : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Definition: [4] Let (F_A, E) be a soft set over U. Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u,e) : e \in A, u \in F_A(e)\}$ which is called a relation form of (F_A, E). Now the **characteristic function** of R_A is written

by,

$$\chi_{R_A} : U \times E \to \{0,1\},\ \chi_{R_A} = \begin{cases} 1, (u,e) \in R_A \\ 0, (u,e) \notin R_A \end{cases}$$

Let $U = \{u_1, u_2, , u_m\}$, $E = \{e_1, e_2, ..., e_n\}$, then R_A can be presented by a table as in the following form

	e_1	e_2	....	e_n
u_1	$\chi_{R_A}(u_1,e_1)$	$\chi_{R_A}(u_1,e_2)$	....	$\chi_{R_A}(u_1,e_n)$
u_2	$\chi_{R_A}(u_2,e_1)$	$\chi_{R_A}(u_2,e_2)$	....	$\chi_{R_A}(u_2,e_n)$
....	....	....	....	....
u_m	$\chi_{R_A}(u_m,e_1)$	$\chi_{R_A}(u_m,e_2)$	....	$\chi_{R_A}(u_m,e_n)$

If $a_{ij} = \chi_{R_A}(u_i, e_j)$, we can define a matrix

$$[a_{ij}]_{m\times n} = \begin{pmatrix} a_{11} & a_{12} & ....a_{1n} \\ a_{21} & a_{22} & ....a_{2n} \\ .... & .... & .... \\ a_{m1} & a_{m2} & ....a_{mn} \end{pmatrix}$$

which is called a **soft matrix** of order $m \times n$ corresponding to the soft set (F_A, E) over U. A soft set (F_A, E) is uniquely characterized by the matrix $[a_{ij}]_{m\times n}$. Therefore we shall identify any soft set with its soft matrix and use these two concepts as interchangeable.

III Some New Concepts of Matrices in Fuzzy Soft Set Theory:

A. Fuzzy Soft Matrix:

Let (F_A, E) be a fuzzy soft set over U. Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u,e) : e \in A, u \in F_A(e)\}$ which is called a relation form of (F_A, E). Now the **characteristic function** of R_A is written by, $\chi_{R_A} : U \times E \to [0,1]$ s.t,

$\chi_{R_A}(u,e) = \mu(u,e)$ [where $\mu(u,e)$ is the membership value of the object u associated with the parameter e.]

Now if the set of universe $U = \{u_1, u_2, \ldots\ldots\ldots\ldots, u_m\}$ and the set of parameters $E = \{e_1, e_2, \ldots\ldots\ldots, e_n\}$, then R_A can be presented by a table as in the following form

	e_1	e_2	…….	e_n
u_1	$\chi_{R_A}(u_1,e_1)$	$\chi_{R_A}(u_1,e_2)$	………	$\chi_{R_A}(u_1,e_n)$
u_2	$\chi_{R_A}(u_2,e_1)$	$\chi_{R_A}(u_2,e_2)$	………	$\chi_{R_A}(u_2,e_n)$
….	………	………	…….	………
u_m	$\chi_{R_A}(u_m,e_1)$	$\chi_{R_A}(u_m,e_2)$	….. ….	$\chi_{R_A}(u_m,e_n)$

If $a_{ij} = \chi_{R_A}(u_i, e_j)$, we can define a matrix

$$[a_{ij}]_{m\times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots\cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots\cdots & a_{2n} \\ \cdots & \cdots & \cdots\cdots\cdots & \cdots \\ a_{m1} & a_{m2} & \cdots\cdots\cdots & a_{mn} \end{pmatrix}$$

which is called a **fuzzy soft matrix** of order $m \times n$ corresponding to the fuzzy soft set (F_A, E) over U. A fuzzy soft set (F_A, E) is uniquely characterized by the matrix $[a_{ij}]_{m\times n}$. Therefore we shall identify any fuzzy soft set with its fuzzy soft matrix and use these two concepts as interchangeable.

Example 1:

Suppose the initial universe set U contains five dresses d_1, d_2, d_3, d_4, d_5 and parameter set

E = { costly, beautiful, cheap, comfortable, gorgeous }= $\{e_1, e_2, e_3, e_4, e_5\}$.

Let $A = \{e_2, e_3, e_4, e_5\} \subset E$ and $F : A \to P(U)$ s.t,

$F(e_1) = \{d_1/0.8, d_2/0.3, d_3/0.6, d_4/0.5, d_5/0.2\}$,

$F(e_2) = \{d_1/0.8, d_2/0.2, d_3/0.5, d_4/0.4, d_5/0.1\}$,

$F(e_3) = \{d_1/0.3, d_2/0.7, d_3/0.5, d_4/0.4, d_5/0.9\}$,

$F(e_4) = \{d_1/0.8, d_2/0.6, d_3/0.4, d_4/0.2, d_5/0.7\}$,

$F(e_5) = \{d_1/0.5, d_2/0.2, d_3/0.8, d_4/0.3\}$

Then we write a fuzzy soft set describing the attractiveness of the dresses is given by,

$(F, A) = \{(e_1, \{d_1/0.8, d_2/0.3, d_3/0.6, d_4/0.5, d_5/0.2\}), (e_2, \{d_1/0.8, d_2/0.2, d_3/0.5, d_4/0.4, d_5/0.1\}), (e_3, \{d_1/0.3, d_2/0.7, d_3/0.5, d_4/0.4, d_5/0.9\}), (e_4, \{d_1/0.8, d_2/0.6, d_3/0.4, d_4/0.2, d_5/0.7\}), (e_5, \{d_1/0.5, d_2/0.2, d_3/0.8, d_4/0.3\})\}$

and then the relation form of (F, A) is written by,

$$R_A = \{(\{d_1/0.8, d_2/0.3, d_3/0.6, d_4/0.5, d_5/0.2\}, e_1),$$
$$(\{d_1/0.8, d_2/0.2, d_3/0.5, d_4/0.4, d_5/0.1\}, e_2),$$
$$(\{d_1/0.3, d_2/0.7, d_3/0.5, d_4/0.4, d_5/0.9\}, e_3),$$
$$(\{d_1/0.8, d_2/0.6, d_3/0.4, d_4/0.2, d_5/0.7\}, e_4),$$
$$(\{d_1/0.5, d_2/0.2, d_3/0.8, d_4/0.3\}, e_5)\}$$

Hence the fuzzy soft matrix (a_{ij}) is written

by, $$(a_{ij}) = \begin{pmatrix} 0.8 & 0.8 & 0.3 & 0.8 & 0.5 \\ 0.3 & 0.2 & 0.7 & 0.6 & 0.2 \\ 0.6 & 0.5 & 0.5 & 0.4 & 0.8 \\ 0.5 & 0.4 & 0.4 & 0.2 & 0.3 \\ 0.2 & 0.1 & 0.9 & 0.7 & 0 \\ & & & & \end{pmatrix}$$

B. Row-Fuzzy Soft Matrix:

A fuzzy soft matrix of order $1 \times n$ i.e., with a single row is called a **row-fuzzy soft matrix**. Physically, a row-fuzzy soft matrix formally corresponds to a fuzzy soft set whose universal set contains only one object.

C. Column-Fuzzy Soft Matrix:

A fuzzy soft matrix of order $m \times 1$ i.e., with a single column is called a **column-fuzzy soft matrix**. Physically, a column-fuzzy soft matrix formally corresponds to a fuzzy soft set whose parameter set contains only one parameter.

D. Square Fuzzy Soft Matrix:

A fuzzy soft matrix of order $m \times n$ is said to be a **square fuzzy soft matrix** if $m = n$ ie., the number of rows and the number of columns are equal. That means a square-fuzzy soft matrix is formally equal to a fuzzy soft set having the same number of objects and parameters.

E. Null Fuzzy Soft Matrix:

A fuzzy soft matrix of order $m \times n$ is said to be a **null fuzzy soft matrix or zero fuzzy soft matrix** if all of its elements are zero. A null fuzzy soft matrix is denoted by, $\tilde{\Phi}$. Now the fuzzy soft set associated with a null fuzzy soft matrix must be a null fuzzy soft set.

F. Complete or Absolute Fuzzy Soft Matrix:

A fuzzy soft matrix of order $m \times n$ is said to be a **complete or absolute fuzzy soft matrix** if all of its elements are one. A complete or absolute fuzzy soft matrix is denoted by, C_A. Now the fuzzy soft set associated with an absolute fuzzy soft matrix must be an absolute fuzzy soft set.

G. Diagonal Fuzzy Soft Matrix:

A square fuzzy soft matrix of order $m \times n$ is said to be a **diagonal-fuzzy soft matrix** if all of its non-diagonal elements are zero.

H. Transpose of a Fuzzy Soft Matrix:

The **transpose** of a square fuzzy soft matrix (a_{ij}) of order $m \times n$ is another square fuzzy soft matrix of order $n \times m$ obtained from (a_{ij}) by interchanging its rows and columns. It is denoted by $(a_{ij})^T$. Therefore the fuzzy soft set associated with $(a_{ij})^T$ becomes a new fuzzy soft set over the same universe and over the same set of parameters.

I. Choice Matrix:

It is a square matrix whose rows and columns both indicate parameters. If ξ is a choice matrix, then its element $\xi(i, j)$ is defined as follows:

$$\xi(i,j) = 1 \text{ when } i-th \text{ and } j-th \text{ parameters are both choice parameters of the decision makers}$$
$$= 0 \text{ otherwise, i.e. when atleast one of the } i-th \text{ or } j-th \text{ parameters be not under choice}$$

There are different types of choice matrices according to the number of decision makers. Like the choice matrices associated with a soft set based decision making problem; here also the choice matrices only contain the digits 0 and 1, the only difference is about the nature of the associated parameters. In previous case the parameters are crisp and here they are fuzzy in nature. We may realize this by the following example.

Example :

Suppose that U be a set of four factories, say,

U = $\{f_1, f_2, f_3, f_4\}$

Let E be a set of parameters, given by,

$E = \{$ *costly, excellent work culture, assured production, good location, cheap* $\}$

= $\{e_1, e_2, e_3, e_4, e_5\}$ (say)

Now let the fuzzy soft set (F, A) describing the quality of the factories, is given by,

$(F,E) = \{$ *costly factories* $= \{f_1/0.9, f_2/0.2, f_3/0.4, f_4/0.8\}$, *factories with excellent work culture* $= \{f_1/0.8, f_2/0.3, f_3/0.5, f_4/0.4\}$, *factories with assured production* $= \{f_1/0.9, f_2/0.2, f_3/0.4, f_4/0.8\}$, *factories with good location* $= \{f_1/0.7, f_2/0.9, f_3/0.4, f_4/0.8\}$, *cheap factories* $= \{f_1/0.1, f_2/0.7, f_3/0.5, f_4/0.2\}\}$

Suppose Mr.X wants to buy a factory on the basis of his choice parameters excellent work culture, assured production and cheap which form a subset P of the parameter set E.

Therefore $P = \{e_2, e_3, e_5\}$

Now **the choice matrix of Mr.X** is,

$$(\xi_{ij})_P = e_P \begin{pmatrix} & & e_P & & \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

Now suppose Mr.X and Mr.Y together wants to buy a factory according to their choice parameters. Let the choice parameter set of Mr.Y be,

$Q = \{e_1, e_2, e_3, e_4\}$

Then **the combined choice matrix of Mr.X and Mr.Y** is

$$(\xi_{ij})_{(P,Q)} = e_P \begin{pmatrix} & & e_Q & & \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

[Here the entries $e_{ij} = 1$ indicates that e_i is a choice parameter of Mr.X and e_j is a choice parameter of Mr.Y. Now $e_{ij} = 0$ indicates either e_i fails to be a choice parameter of Mr.X or e_j fails to be a choice parameter of Mr.Y.]

Again the above combined choice matrix of Mr.X and Mr.Y may be also presented in its transpose form as,

$$(\xi_{ij})_{(Q,P)} = e_Q \begin{pmatrix} & & e_P & & \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Now let us see the form of the combined choice matrix associated with three decision makers. Suppose that Mr.Z is willing to buy a factory together with Mr.X and Mr.Y on the basis of his choice parameters excellent work culture, assured production and good location which form a subset R of the parameter set E.

Therefore $R = \{e_2, e_3, e_4\}$

Then **the combined choice matrix of Mr.X, Mr.Y and Mr.Z** will be of three different types which are as follows,

i)

$$(\xi_{ij})_{(R,P\wedge Q)} = e_R \begin{pmatrix} & & e_{(P\wedge Q)} & & \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$[\textit{Since the set of common choice parameters of } Mr.X \textit{ and } Mr.Y \textit{ is}, P \wedge Q = \{e_2, e_3\}]$

[Here the entries $e_{ij} = 1$ indicates that e_i is a choice parameter of Mr.Z and e_j is a common choice parameter of Mr.X, Mr.Y. Now $e_{ij} = 0$ indicates either e_i fails to be a choice parameter of Mr.Z or e_j fails to be a common choice parameter of Mr.X and Mr.Y.]

ii) $$(\xi_{ij})_{(P,Q\wedge R)} = e_P \begin{pmatrix} & & e_{(Q\wedge R)} & & \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$[\textit{ Since } Q \wedge R = \{e_2, e_3, e_4\}]$

iii)

$$(\xi_{ij})_{(Q,R\wedge P)} = e_Q \begin{pmatrix} & & e_{(R\wedge P)} & & \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$[\textit{ Since } R \wedge P = \{e_2, e_3\}]$

J. Symmetric Fuzzy Soft Matrix:

A square fuzzy soft matrix A of order $n \times n$ is said to be a **symmetric fuzzy soft matrix**, if its transpose be equal to it, i.e., if $A^T = A$. Hence the fuzzy soft matrix (a_{ij}) is symmetric, if $a_{ij} = a_{ji}, \forall i, j$.

Therefore if (a_{ij}) be a symmetric fuzzy soft matrix then the fuzzy soft sets associated with (a_{ij}) and $(a_{ij})^T$ both be the same.

K. Addition of Fuzzy Soft Matrices:

Two fuzzy soft matrices A and B are said to be conformable for addition, if they be of the same order. The sum of two fuzzy soft matrices A and B of the same order is the fuzzy soft matrix whose elements are taken as the maximum element of the corresponding elements of the two fuzzy soft matrices A and B.

Therefore the addition of two fuzzy soft matrices (a_{ij}) and (b_{ij}) of order $m \times n$ is defined by,

$(a_{ij}) \oplus (b_{ij}) = (c_{ij})$, where (c_{ij}) is also an $m \times n$ fuzzy soft matrix and $c_{ij} = max\{a_{ij}, b_{ij}\} \forall i, j$.

L. Subtraction of Fuzzy Soft Matrices: Two fuzzy soft matrices A and B are said to be conformable for subtraction, if they be of the same order. The subtraction of a fuzzy soft matrix B from a fuzzy soft matrix A is a fuzzy soft matrix whose elements are taken as the minimum element of the corresponding elements of the two fuzzy soft matrices A and B^o (where B^o is the complement of B).

Therefore for any two fuzzy soft matrices (a_{ij}) and (b_{ij}) of order $m \times n$, the subtraction of (b_{ij}) from (a_{ij}) is defined as,

$(a_{ij})!(b_{ij}) = (c_{ij})$, where (c_{ij}) is also an $m \times n$ fuzzy soft matrix and $c_{ij} = min\{a_{ij}, b^o_{ij}\} \forall i, j$.

Properties: Let A be a fuzzy soft matrix of order $m \times n$. Then

i) $A \oplus A^o = C_A$

ii) $A ! A = \tilde{\Phi}$

M. Product of a Fuzzy Soft Matrix with a Choice Matrix: Let U be the set of universe and E be the set of parameters. Suppose that A be any fuzzy soft matrix and B be any choice matrix of a decision maker concerned with the same universe U and E. Now if the number of columns of the fuzzy soft matrix A be equal to the number of rows of the choice matrix B, then A and B are said to be conformable for the product ($A \otimes B$) and the product ($A \otimes B$) becomes a fuzzy soft matrix. We may denote the product by $A \otimes B$ or simply by AB.

If $A = (a_{ij})_{m \times n}$ and $B = (b_{jk})_{n \times p}$, then

$$AB = \begin{cases} (c_{ik})_{m \times p} \ \ if \ \ 0 \le c_{ik} \le 1 \forall i = 1,2,...,m; \\ k = 1,2,...,p \\ \\ the\ normalized\ form\ of\ (c_{ik})_{m \times p}\ \ if \\ c_{ik} > 1\ for\ at\ least\ one\ i(=1,2,...,m) \\ or, k(=1,2,...,p) \end{cases}$$

where $c_{ik} = \sum_{j=1}^{n} min\{a_{ij}, b_{jk}\}$ and normalize the array $(c_{ik})_{m \times p}$ by dividing each entry of the array by the sum of the all entries of the array.

Example:

Let U be the set of four dresses, given by, U = $\{d_1, d_2, d_3, d_4\}$. Let E be the set of parameters, given by, E = $\{\ cheap, beautiful, comfortable, gorgeous\ \} = \{e_1, e_2, e_3, e_4\}$ (say). Suppose that the set of choice parameters of Mr.X be, $A = \{e_1, e_3\}$. Now let according to the choice parameters of Mr.X, we have the fuzzy soft set (F, A) which describes the attractiveness of the dresses and the fuzzy soft matrix of the fuzzy soft set (F, A) be,

$$(a_{ij}) = \begin{pmatrix} 0.8 & 0.2 & 0.7 & 0.3 \\ 0.3 & 0.7 & 0.4 & 0.8 \\ 0.7 & 0.4 & 0.5 & 0.6 \\ 0.5 & 0.1 & 0.9 & 0.2 \end{pmatrix}$$

Again the choice matrix of Mr.X is,

$$(\xi_{ij})_A = e_A \begin{pmatrix} & e_A & & \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Since the number of columns of the fuzzy soft matrix (a_{ij}) is equal to the number of rows of the choice matrix $(\xi_{ij})_A$, they are conformable for the product.

Therefore

$$\begin{pmatrix} 0.8 & 0.2 & 0.7 & 0.3 \\ 0.3 & 0.7 & 0.4 & 0.8 \\ 0.7 & 0.4 & 0.5 & 0.6 \\ 0.5 & 0.1 & 0.9 & 0.2 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \textit{the normalized form of} \begin{pmatrix} 1.5 & 0 & 1.5 & 0 \\ 0.7 & 0 & 0.7 & 0 \\ 1.2 & 0 & 1.2 & 0 \\ 1.4 & 0 & 1.4 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1.5/9.6 & 0/9.6 & 1.5/9.6 & 0/9.6 \\ 0.7/9.6 & 0/9.6 & 0.7/9.6 & 0/9.6 \\ 1.2/9.6 & 0/9.6 & 1.2/9.6 & 0/9.6 \\ 1.4/9.6 & 0/9.6 & 1.4/9.6 & 0/9.6 \end{pmatrix} [$$

Since 9.6 is the sum of all elements of the previous array]

$$= \begin{pmatrix} 0.16 & 0 & 0.16 & 0 \\ 0.07 & 0 & 0.07 & 0 \\ 0.13 & 0 & 0.13 & 0 \\ 0.15 & 0 & 0.15 & 0 \end{pmatrix}$$

IV Algorithm:

This new approach is specially based on choice matrices. These choice matrices represent the choice parameters of the decision makers and also help us to solve the fuzzy soft matrix based decision making problems with least computational complexity. So by the help of these newly proposed choice matrices and the operations on them we are presenting the following algorithm:

Step-1: First construct the combined choice matrix with respect to the choice parameters of the decision makers.

Step-2: Compute the product fuzzy soft matrices by multiplying each given fuzzy soft matrix with the combined choice matrix as per the rule of multiplication of fuzzy soft matrices.

Step-3: Compute the sum of these product fuzzy soft matrices to have the resultant fuzzy soft matrix(R_f).

Step-4: Then compute the weight of each object(O_i) by adding the entries of its concerned row(i-th row) of R_f and denote it as $W(O_i)$.

Step-5: The object having the highest weight becomes the optimal choice object. If more than one object have the highest weight then any one of them may be chosen as the optimal choice object.

To illustrate the basic idea of the algorithm, now we apply it to a fuzzy soft set based decision making problem.

Example:

Let the set of universe U consist of three ponds P_1, P_2, P_3 and the set of parameters

$$E = \{\ \textit{area, amount of water, quality of water, commercial benifit}\ \} = \{e_1, e_2, e_3, e_4\}$$

.

Now two friends Ram and Shyam together wants to buy a pond among these three ponds. Ram is purchasing the pond for his personal use and Shyam is interested for business purpose. So the sets of choice parameters of Ram and Shyam are respectively,

$A = \{e_2, e_3\} \subset E$ and

$B = \{e_2, e_4\} \subset E$

Now let according to the choice parameters of Ram and Shyam, we have the fuzzy soft sets (F, A) and (G, B), both describing the importance of the ponds according to Ram and Shyam respectively. Let the fuzzy soft matrices of the fuzzy soft sets (F, A) and (G, B) are respectively,

$$(a_{ij}) = \begin{pmatrix} 0 & 0.9 & 0.5 & 0 \\ 0 & 0.6 & 0.8 & 0 \\ 0 & 0.5 & 0.4 & 0 \end{pmatrix},$$

$$(b_{ik}) = \begin{pmatrix} 0 & 0.8 & 0 & 0.6 \\ 0 & 0.5 & 0 & 0.7 \\ 0 & 0.6 & 0 & 0.8 \end{pmatrix}$$

Now the problem is to select the pond among the three ponds which satisfies the choice parameters of Ram and Shyam as much as possible.

1) The combined choice matrix of Ram and Shyam is,

$$e_B \begin{pmatrix} & & e_A & \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

or it may be presented in its transpose form as,

$$e_A \begin{pmatrix} & & e_B & \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2) Corresponding product fuzzy soft matrices are,

$$U_A \begin{pmatrix} & & e_A & \\ 0 & 0.9 & 0.5 & 0 \\ 0 & 0.6 & 0.8 & 0 \\ 0 & 0.5 & 0.4 & 0 \end{pmatrix} \otimes$$

$$e_A \begin{pmatrix} & & e_B & \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ = the normalized form of

$$\begin{pmatrix} 0 & 1.4 & 0 & 1.4 \\ 0 & 1.4 & 0 & 1.4 \\ 0 & 0.9 & 0 & 0.9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0.37 & 0 & 0.37 \\ 0 & 0.37 & 0 & 0.37 \\ 0 & 0.24 & 0 & 0.24 \end{pmatrix}$$

$$U_B \begin{pmatrix} & & e_B & \\ 0 & 0.8 & 0 & 0.6 \\ 0 & 0.5 & 0 & 0.7 \\ 0 & 0.6 & 0 & 0.8 \end{pmatrix} \otimes$$

$$e_B \begin{pmatrix} & & e_A & \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$ = the normalized form of

$$\begin{pmatrix} 0 & 1.4 & 1.4 & 0 \\ 0 & 1.2 & 1.2 & 0 \\ 0 & 1.4 & 1.4 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0.35 & 0.35 & 0 \\ 0 & 0.30 & 0.30 & 0 \\ 0 & 0.35 & 0.35 & 0 \end{pmatrix}$$

3) The sum of these product fuzzy soft matrices is,

$$\begin{pmatrix} 0 & 0.37 & 0 & 0.37 \\ 0 & 0.37 & 0 & 0.37 \\ 0 & 0.24 & 0 & 0.24 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0.35 & 0.35 & 0 \\ 0 & 0.30 & 0.30 & 0 \\ 0 & 0.35 & 0.35 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0.37 & 0.35 & 0.37 \\ 0 & 0.37 & 0.30 & 0.37 \\ 0 & 0.35 & 0.35 & 0.24 \end{pmatrix} = R_f$$

4)Now the weights of the ponds are,

- $W(P_1) = 0 + 0.37 + 0.35 + 0.37 = 1.09$
- $W(P_2) = 0 + 0.37 + 0.30 + 0.37 = 1.04$
- $W(P_3) = 0 + 0.35 + 0.35 + 0.24 = 0.94$

5) The pond associated with the first row of the resultant fuzzy soft matrix(R_f) has the highest weight $(W(P_1) = 1.09)$, therefore P_1 be the optimal choice pond. Hence Ram and Shyam will buy the pond P_1 according to their choice parameters.

V Conclusion:

In this paper first we have defined different types of fuzzy soft matrices and then introduced some operations on them. Moreover we have proposed the concept of choice matrix which represents the choice parameters of the decision makers associated with a fuzzy soft set based decision making problem. Finally we have presented a new algorithm using these choice matrices and newly proposed operations of fuzzy soft matrices to solve fuzzy soft set based decision making problems. The speciality of this new method is that it may solve any fuzzy soft set based decision making problem involving huge number of decision makers very easily along with a simple computational procedure.

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