Computer Aided Design and Analysis of Ring Stiffened Cylindrical Shell for Underwater Applications

Pulletikurti V Manikanta[1]
P. Ranga Raju[2]
[1]M Tech (Machine Design) II Year II Semester
[2] Associate Professor
Department of Mechanical Engineering, MVGR College of Engineering, Andhra Pradesh, India

Y. Appa Rao[3]
[3]Scientist-E, Thermal Propulsion Division, N.S.TL, Visakhapatnam, Andhra Pradesh, India

***Abstract*: Underwater weapons like torpedoes, mines etc. are used in defence applications. These are designed for moderate and extreme depths require minimization of structural weight for increasing the performance, speed and operating range.**

The present paper deals with the design and analysis of ring stiffened cylindrical shell which is made of Aluminium alloy forging. This shell will be used for accommodating the electronic systems and devices required for underwater applications. It will be subjected to external hydrostatic pressure during its functioning.

In this paper the design and analysis of the underwater weapon shell is carried out using analytical techniques, empirical codes such as A.S.M.E and PD 5500 and ANSYS Finite Element Analysis software. The design and analysis of the shell structure is mainly focused to obtain stresses and buckling load factors.

***Keywords*: External Pressure, Shells, Yield Failure, Buckling Failure, FEA etc.**

I. INTRODUCTION

Shell structures play an important role for underwater applications, space vehicles and aircrafts in terms of its buckling, stiffness, strength and weight etc. Also the importance of exploration of lightweight and high strength materials such as aluminium alloys, titanium alloys and composite materials have become significant.

Though metals have given excellent service for the construction of shells in hydrospace applications, the requirements for service at greater depths has created considerable difficulties with metal owing to the problems of weight. A point is reached where the shell becomes so thick that all buoyancy is lost and the payload is reduced to zero. Always there is a need for design of shell structures for underwater applications like torpedoes, submarines, mines etc. for their improved performance. These are designed for moderate and extreme depths require minimization of structural weight for increasing performance, speed and operating range. As a result, interest is being shown in lightweight materials such as aluminum and composite materials for the above underwater applications.

Modes of failure of shells subjected to external pressure:

When a shell is subjected to external pressure, failure may occur in two modes, they are

A. Yield failure

Failure due to yield is perhaps the most important modes of failure of vessels under external pressure. Shell yield is analogous to the behavior of a short stubby column. It depends on yield stress. When the shell is relatively heavy and the frame spacing is close, the shell will fail in yield. A sensible design should eliminate failure due to instability. This is because of the difficulty of predicting the loss in buckling resistance of a shell due to the detrimental effects of its initial geometric imperfections such imperfections in a vessel under external water pressure can cause a catastrophic fall in its buckling resistance.

B. Buckling failure

Buckling occurs when a member or a structure converts its membrane strain energy into strain energy of bending with no change in externally applied loads. A critical condition, at which the buckling impends, exists when it is possible that the deformation state may change slightly in a way that makes loss in membrane strain energy equal to gain in bending strain energy. When cylindrical or conical shells are subjected to external pressures and the primary stresses become compressive then the shell is said to be subjected to the phenomenon called buckling. Instability in the shell occurs well below the yield strength of the material of the shell. Thin cylinders are those in which the thickness of the cylinders is smaller when compared to its diameter so that the elastic buckling occurs prior to the wall material reaching its yield strength. Experiments shows when load on the shell reaches a critical value, radial deflections begin to impend rapidly and the shell completely fails. The significant characteristic of the buckling is that the deflections are not proportional to the load applied on the shell.

1. Shell Instability of Ring-Stiffened Circular Cylinders

Under uniform external pressure, a thin-walled circular cylinder may buckle, usually at a fraction of that pressure required to cause axi symmetric yield. If the circular cylinder is very long, buckling resistance will be very small, the vessel suffering from flattening mode (i.e. ovalling) [9]. The shell instability is shown in Fig. 1

Fig.1 Shell instability

2. General Instability of Ring-Stiffened Circular Cylinders (Overall buckling)

General instability is the process where the entire ring-shell combination of a circular cylinder buckles bodily between adjacent bulk heads. This mode of failure was first identified by Tokugawa in 1929. The general instability is shown in Fig.2

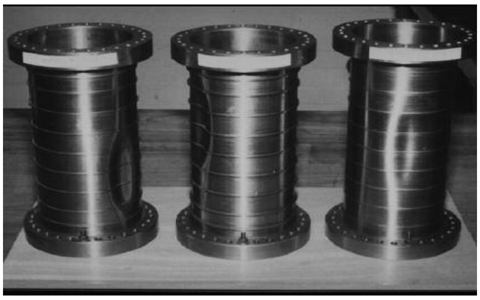

Fig. 2 General instability

II. DESIGN METHODOLOGY

A. Material Properties

The mechanical and physical properties of HF-15, (Al-cu alloy) are given below:

- Tensile strength : 435 N/mm^2
- Yield strength : 385 N/mm^2
- Elongation : 6 %

- Young's modulus : 70000 N/mm^2
- Poisson's ratio : 0.33
- Density : 2700 $\frac{kg}{m^3}$

B. Design of Thin Shell

According to PD 5500 (Unfired Fusion Welded Pressure Vessels), [4]

If the wall thickness of the shell (t) is less than 1/10th of radius of the shell is called thin shell.

$\therefore t < \frac{1}{10}R = 8 < \frac{1}{10} \times 266.5 \Rightarrow 8 < 26.65$. So, the considering shell is Thin shell.

C. Stiffener Design

Stiffener A structural member attached to the skin to resist compressive and bending loads. It also helps to hold the shape of the shell wings and other surfaces.

Spacing between stiffeners (L_s)

According to ASME codes, Section-8, Division-1,

The maximum clear spacing between stiffeners of '8t' for external Rings and '12t' for internal Rings, where t' is the shell thickness at the attachment [2].

So, for internal rings (L_s) = 12 $\times t$ = 96 mm.

Considered General Instability for overall buckling of stiffeners and shell at a time so, the design procedure as per following.

D. Design against general elastic instability (p_n) *(Overall buckling)*

p_n For values of n = 2,3,4,5 and 6 using:

$$p_n = \frac{Et\beta}{R} + \frac{(n^2 - 1)}{R^3 L_s} EI_c$$

This is the Bryant's formula to find out the elastic Buckling pressure. [6]

Where,

$$\beta = \frac{1}{\left\{n^2 - 1 + 0.5\left(\frac{\pi R}{L_s}\right)^2\right\}\left\{n^2\left(\frac{L_s}{\pi R}\right)^2 + 1\right\}^2}$$

$$I_c = \frac{t^3 L_e}{3} + I_s + A_s\left[\frac{t}{2} + \lambda(R - R_s)\right]^2 - A_c X_c^2$$

$$I_s = \frac{bd^3}{12}$$

$$X_c = \left\{\frac{t^2}{2}L_e + A_s\left[\frac{t}{2} + \lambda(R - R_s)\right]\right\}/A_c$$

λ = parameter for internal stiffener = +1

$$L_e = \frac{A\sqrt{Rt}}{\sqrt{\sqrt{1 + B_1 x^2} + Cx}}$$

This is the Biljaard's expression to find out the Effective length of shell acting with stiffener [3].

Where,

$$x = \frac{n^2 \times \frac{t}{R}}{}$$

$$u = \frac{\frac{L_s}{R}}{\sqrt{\frac{t}{R}}}$$

A = u/ (1/1.098+0.03u^3)

B1 = u-1

C = 0.6$\times$ (1-0.27u)u^2

A_c = (A_s +t L_e)

n	B	L_e (mm)	A_c (mm^2)	X_c	I_c (mm^4)	p_n $\left(\frac{N}{mm^2}\right)$
2	6.3$\times 10^{-3}$	75.51	856.1	7.82	40166	18.3
3	5.9$\times 10^{-4}$	69.52	808.2	8.05	39080	**13.8**
4	1.0$\times 10^{-4}$	62.41	751.3	8.36	37770	23.0
5	2.8$\times 10^{-5}$	55.17	693.3	8.72	36268	35.1
6	9.7$\times 10^{-6}$	48.54	640.37	9.11	34704	48.9

Table 1 Comparison of Buckling parameters

Checked that n = 2,3,4,5 and 6, $P_n \geq kp$, so the design against elastic instability is safe [4]. The comparison of buckling parameters are shown in Table 1

Where,

k = Stiffener fabrication factor = 1.8 for hot formed stiffener

p = Applied pressure = 5 $\frac{N}{mm^2}$

The least value of Elastic Instability P_n has taken to compare with Finite Element Analysis value because the buckling starts from that value of pressure. The plot between Buckling pressures and Number of Lobes is shown in Fig. 3

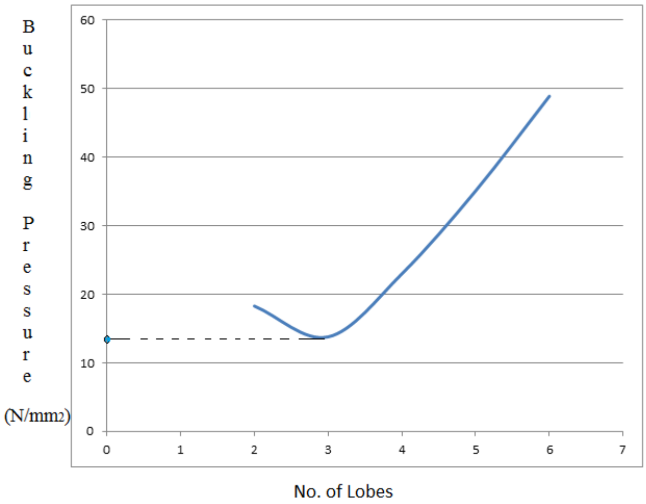

Fig. 3 Plot between Buckling Pressures and No. of Lobes

E. Design of moment of inertia of ring-shell combination

According to ASME codes, [2]

a. Approximate moment of inertia (I)

$$I = \frac{0.16\, D^3\, p\, L_s}{E}$$

$$= \frac{0.16 \times 533^3 \times 5 \times 96}{70000}$$

= 166128.7 mm^4

b. Required moment of inertia (I_R)

$$I_R = \frac{D\, L_s \left(t + \frac{A_s}{L_s}\right) A_{FAC}}{10}.9$$

$$= \frac{533 \times 96 \times \left(8 + \frac{252}{96}\right) \times 5.3747 \times 10^{-3}}{10.9}$$

= 142883.5 mm^4

Where,

$$\text{FAC 'A'} = \frac{2 \times B_{FAC}}{E}$$

= 5.37$\times 10^{-3}$

$$\text{FAC 'B'} = 0.75 \times \frac{pD}{\left(t + \frac{A_s}{L_s}\right)}$$

= 188.11

$\therefore I_R < I$,

The Required moment of inertia is less than the approximate moment of inertia, so the design is satisfactory.

F. Axi symmetric Yield Failure of Ring-Stiffened circular cylinders

One of the earliest solutions presented for the axi symmetric deformation of circular cylinders was that due to Von Sanden & Gunther in 1920 [20], based on the differential equation:

$$\frac{d^4w}{dx^4} + \frac{12(1-\vartheta^2)w}{t^2R^2} = \frac{12(1-\vartheta^2)p}{Et^3} \quad (1.1)$$

A smaller error with the Von Sanden & Gunther [20], solution is that their differential equation did not fully take into account the loading on the shell caused by the pressure normal to it, the so-called Viterbo effect. The Viterbo effect, however, is only about 1% and its inclusion in the differential equation (1.1)

$$\frac{d^4w}{dx^4} + \frac{12(1-\vartheta^2)w}{t^2R^2} = \frac{12(1-\vartheta^2)p\left(1-\frac{\vartheta}{2}\right)}{Et^3} \quad (1.2)$$

Another deficiency with both (1.1) and (1.2) is that they do not include the beam-column effect, which causes the deformation to be non-linear, and this can be quite large for certain cases. Salerno & Pulos (13) introduced the beam-column effect in the original differential equation, which is shown in equation (1.3). The beam-column effect can increase the maximum longitudinal stress by about 10% for many vessels:

$$\frac{d^4w}{dx^4} + \frac{6(1-\vartheta^2)pR}{Et^3}\frac{d^2w}{dx^2} + \frac{12(1-\vartheta^2)w}{t^2R^2} = \frac{12(1-\vartheta^2)p\left(1-\frac{\vartheta}{2}\right)}{Et^3} \quad (1.3)$$

A further improvement to the differential equation (1.3) was made by Wilson [21], when he solved the differential equation (1.4) using a Fourier cosine transformation:

$$\frac{d^4w}{dx^4} + \left[\frac{\vartheta}{R^2} + \frac{6(1-\vartheta^2)pR}{Et^3}\right]\frac{d^2w}{dx^2} + \frac{12(1-\vartheta^2)w}{t^2R^2} = \frac{12(1-\vartheta^2)p\left(1-\frac{\vartheta}{2}\right)}{Et^3} \quad (1.4)$$

Like Salerno & Pulos [14], Wilson [21], solved this differential equation for a shell stiffened by equal-size ring-stiffeners where the shell deformed symmetrically about mid-span as shown in Fig. 4.The boundary conditions assumed by Wilson were:

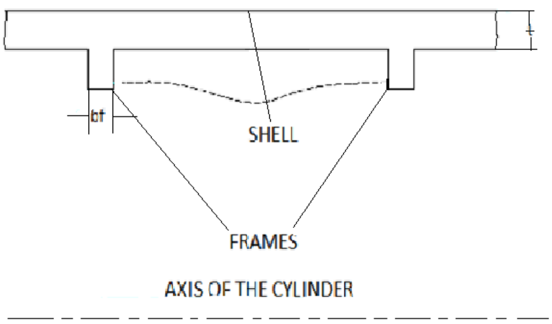

Fig. 4 Cross section of Ring Stiffened cylindrical shell

a) w is symmetrical about x = 0 (mid span);

b) $\frac{dw}{dx}$ = 0 at x = ± L/2; (1.5)

c) $[(Et^3)/(6(1-$ $= G_1 w - H_1 1]$ at x = ± L/2

Where,

$$G_1 = E\left(\frac{A_s}{a_f^2} + \frac{bt}{R^2}\right);$$

$$H_1 = pb\left(1-\frac{\vartheta}{2}\right);$$

Ross [9], solved Wilson's [21], differential equation using the method of Salerno & Pulos [15], by putting

a) $\alpha^4 = \frac{3(1-\vartheta^2)}{R^2t^2}$ $\beta^2 = \frac{pR^3}{2Et} + \frac{t^2\vartheta}{12(1-\vartheta^2)}$;

b) $C_o = pR^2 (1-\vartheta/2)/(Et)$.

To give the differential equation (1.6):

$$\frac{d^4w}{dx^4} + 4\alpha^4\beta^2\frac{d^2w}{dx^2} + 4\alpha^4 w = \frac{12(1-\vartheta^2)C_o}{t^2R^2} \quad (1.6)$$

Putting

$$F_1 = \alpha\sqrt{(1-\alpha^2\beta^2)};\; F_2 = \alpha\sqrt{(1+\alpha^2\beta^2)}$$

The complete solution of (1.6) is

$w=(A_1\cosh F_1x \times \cos F_2x)+(A_2\sinh F_1x \times \sin F_2x)+(A_3\cosh F_1x \times \sin F_2x)+ (A_4\sinh F_1x \times \cos F_2x) + C_o$ (1.7)

And some of its derivatives are

$\frac{dw}{dx} =((A_1F_1+A_2F_2)\sinh F_1x\cos F_2x)+((A_2F_1+A_1F_2)\cosh F_1x\sin F_2x)+((A_3F_2+A_4F_1)\cosh F_1x\cos F_2x)+((A_3F_1+ A_4F_2)\sinh F_1x\cos F_2x)$ (1.8)

$\frac{d^2w}{dx^2}$ = ([A_1 (F_1^2-F_2^2) +2$A_2F_1F_2$] coshF_1xcosF_2x) + ([A_2 (F_1^2-F_2^2) -2$A_1F_1F_2$]sinhF_1xsinF_2x)+([A_3(F_1^2-F_2^2)-2$A_4F_1F_2$]coshF_1xsinF_2x)+([A_4(F_1^2-F_2^2)+2$A_3F_1F_2$]sinhF_1xcosF_2x) (1.9)

$\frac{d^3w}{dx^3}$ = ([A_1F_1 (F_1^2-3F_2^2) +A_2F_2 (3F_1^2-F_2^2)] sinhF_1xcosF_2x)
+ ([A_1F_2 (F_2^2-3F_1^2) +A_2F_1 (F_1^2-3F_2^2)] coshF_1xsinF_2x)
+ ([A_3F_2 (3F_1^2-F_2^2) +A_4F_1 (F_1^2-3F_2^2)] coshF_1xcosF_2x)
+ ([A_3F_1 (F_1^2-3F_2^2) +A_4F_2 (F_2^2-3F_1^2)] sinhF_1xsinF_2x) (1.10)

The only unknown parts of the derivations shown above are the arbitrary constants, and these can be solved by assuming certain conditions to exist at the boundary, as described earlier in equation (1.5).

For the circular cylindrical shell element, stiffened by equal-strength frames, it can be seen from boundary condition (a) above that the animetric terms must vanish because of the symmetry of w about mid-span, i.e.

$$A_3 = A_4 = 0$$

Thus there are only two unknowns in equation (1.7), namely A_1 and A_2, and these are obtained from conditions (b) and (c) as follows:

$A_1 = N_1/D$, $A_2 = N_2/D$

Where,

N_1=-(G_1C_0-H_1)× (F_1cosh0.5F_1Lsin0.5F_2L+F_2sinh0.5F_1Lcos0.5F_2L);

N_1=(G_1C_0-H_1)× (F_1sinh0.5F_1Lcos0.5F_2L-F_2cosh0.5F_1Lsin0.5F_2L);

D = {$(Et^3)/(12(1-\vartheta^2))$ } {2F1F2 (F_1^2+F_2^2) (coshF_1L-cosF_2L)} + 0.5G_1(F_1sinF_2L+F_2sinhF_1L)

The stress distributions across the bay can be obtained by substituting A_1 and A_2 into equations (1.7) and (1.9), and then by substituting (1.7) and (1.9) into equations (1.11) and (1.12)

$$\text{Hoop Stress}=-\frac{Ew}{R} - \frac{pR\vartheta}{2t} \pm \frac{Et}{2(1-\vartheta^2)}\left(\frac{w}{R^2} + \vartheta\frac{d^2w}{dx^2}\right) \quad (1.11)$$

$$\text{Longitudinal stress}=-\frac{pR}{2t} \pm \frac{Et}{2(1-\vartheta^2)}\left(\frac{\vartheta w}{R^2} + \frac{d^2w}{dx^2}\right) \quad (1.12)$$

III. FINITE ELEMENT ANALYSIS

A. Introduction

The Finite Element Method is firmly established as a powerful and popular analysis tool. It is a numerical procedure for analyzing structures and continua. The Finite Element procedure produces many simultaneous algebraic equations, which are generated and solved on a digital computer. The Finite Element Method originated as a method of stress analysis. Today Finite Element Methods are also used to analyze problems of heat transfer, fluid flow, lubrication, electric and magnetic fields. Finite element procedures are used in design of buildings, electric motors, heat engines, ships, air frames and space craft. The process of subdividing all systems into their individual components or elements whose behavior is readily understood and then rebuilding the original system from such components to study its behavior is a natural way in which the engineer, the scientist or even the economist proceeds. The Finite Element Method, in general, models a structure as an assemblage of small elements. Each element is of simple geometry and therefore is much easier to analyze than the actual structure. ANSYS software is used for carrying out the F.E.A of shell structures.

The shell is modeled using CATIA software as shown in Fig.5

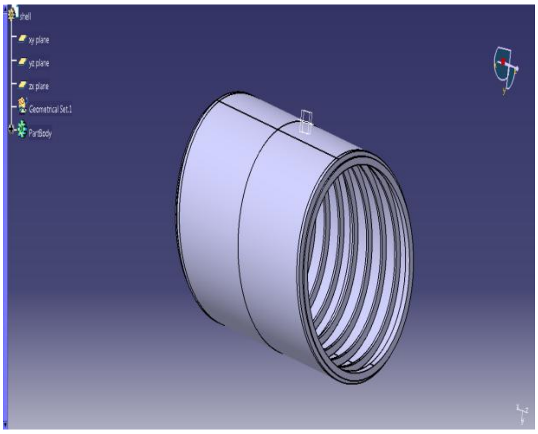

Fig. 5 Solid Modal of Shell structure

B. Preprocessing

Defining the problem, the major steps in preprocessing are given below

- Define key points/lines/areas/volumes
- Define element type and material/geometric properties
- Mesh lines/areas/volumes as required

1. FE Model

ANSYS software is used for meshing the shell structure using shell element (shell93) which is an 8-noded quadrilateral element. The construction is an area model with thickness of each area defined. Also the thickness of the shell and stiffeners are overlapping at their joining edges [1]. The Shell structure before and after meshing is shown Fig. 6 and Fig. 7 respectively. The total number of Nodes and Elements for the Model are 65264 and 20768 respectively.

2. Boundary conditions

The element used has six nodal DOF (Three translations UX, UY, UZ and three rotations ROTX, ROTY, ROTZ). All the nodes along the circumference of the front end and rear end of the shell structure are constrained as UX=UY=UZ =0 &RX= RY= RZ =0

Loading Conditions

A pressure of **50**bar has been applied circumferentially on the external areas of the shell.

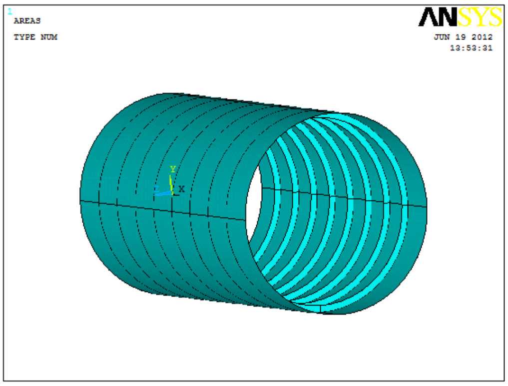

Fig. 6 Shell structure before Meshing

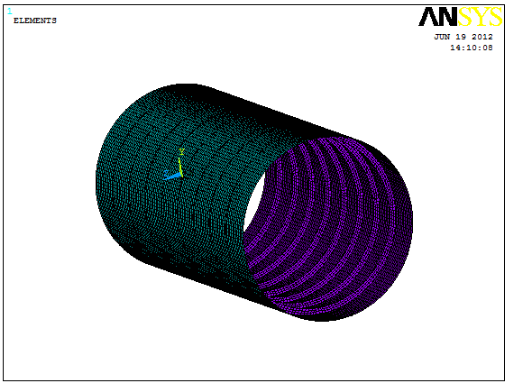

Fig. 7 Shell structure after Meshing

C. Solution

The static and buckling analyses are carried out using ANSYS software solvers to obtain the deformations, stresses and Buckling load factors etc. The methods used for solving static and buckling analysis are Sparse direct and Subspace iteration respectively.

D. Post processing

Post processing of shell structure is carried out for obtaining the results as follows

- Nodal/Element displacement plots
- Nodal/Element stress plots
- Buckling load factors and mode shapes

The Buckling mode of shell structure is shown in Fig. 8 and 9. The Von Mises stress contour plot of shell structure is shown in Fig. 10.

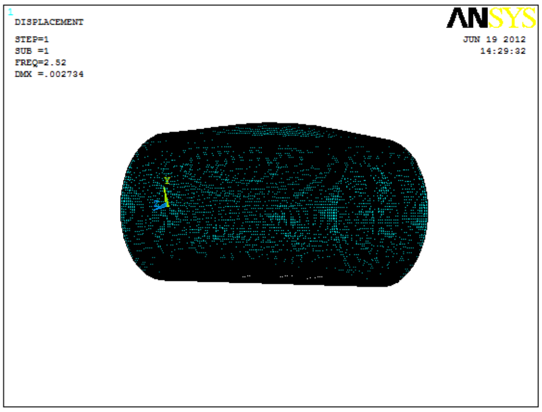

Fig. 8 Deformation Shape by Buckling

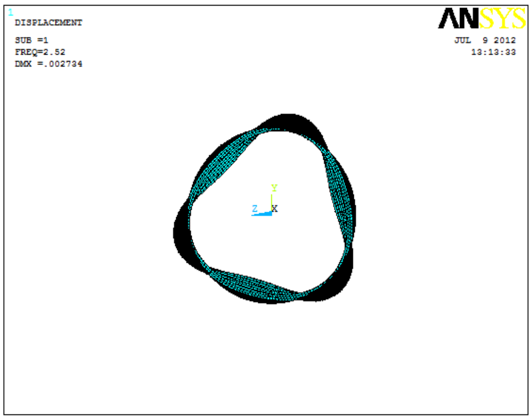

Fig. 9 Mode of Buckle by Circumferential Pressure

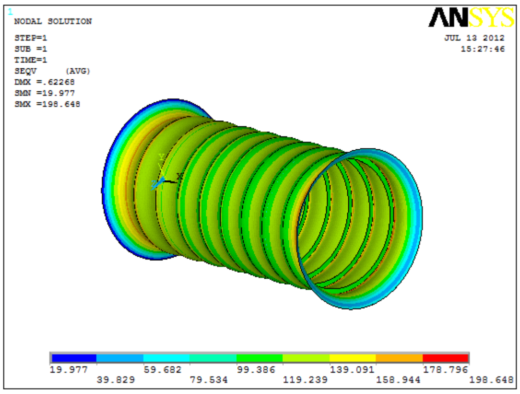

Fig. 10 Von Mises Stress contour of shell structure

IV. RESULTS AND DISCUSSIONS

The Design of shell structure is carried out using ASME, PD5500 codes, C.T.F.ROSS and other analytical approaches for finding out Circumferential stresses and Buckling load factors.

The Designed value against general elastic instability $(p_1 n)$ (Overall buckling)obtained by Bryant's formula to find out the elastic Buckling pressure [6] is compared and found to be in close agreement with the values obtained by Finite Element Analysis.

The Axi symmetric Yield Failure of Ring-Stiffened circular cylinders is obtained from the Equation solved by C.T.F Ross [10] is compared with Finite Element Analysis.

The Comparison of Analytical and Finite Element Analysis values are shown in Table 2.

Description	Von-Mises Stress (N/mm^2)	Buckling Load Factor
Analytical	148.7(Compression)	2.76
FEA	139.1(Compression)	2.52
Allowable	385/1.5= 256.7	>1

Table 2 Comparison of Analytical and F.E.A values

REFERENCES

[1] ANSYS user guide.

[2] A.S.M.E Boiler Pressure Vessel codes, section viii, division 1&2, 2010.

[3] Biljaard, P.P., 'Buckling under External Pressure of cylindrical shells Evenly Stiffened by Rings only', Journal of Aerospace sciences, June 1957.

[4] British Standards Institution, BS 5500 (now superseded by PD 5500) 'Unfired Fusion Welded Pressure Vessels'. HMSO, London, 1991.

[5] Bryan. G.H., Application of the Energy Test to the Collapse of a Long Thin Pipe under External Pressure, Cambridge Philosophical Society Proceedings, 1888.

[6] Bryant A.R. 'Hydrostatic Pressure Buckling of a ring stiffened cylinder' Rep, NCRER306, Dun form line, Scotland, Oct.1954.

[7] C.T.F. Ross, 'Advanced Applied Stress Analysis' Horwood, 1987.

[8] C.T.F. Ross, 'Applied Stress Analysis' Horwood, 1987.

[9] C.T.F. Ross, 'Axi symmetric Deformation of a Varying Strength Ring-Stiffened Cylinder under Lateral and Axial pressure'. Trans, RINA, 112, 1970.

[10] C.T.F. Ross, 'The Collapse of Ring-Reinforced Circular Cylinders under Uniform External Pressure, Manchester University, 1963.

[11] Donell, L.H., 'A New theory for the Buckling of Thin Cylinder under Axial compression and Bending' America society of Mechanical Engineers, vol. 56, 1934.

[12] Flugge, W., 'Stresses in Shells', Springer-Verlag, Berlin, 1962

[13] Koiter, W.T., Elishakoff. I. Li, Y.W, 'Buckling of an Axially Compressed Cylindrical shells of variable thickness, International journal of solids and structures, vol.31, No.6, 1994.

[14] Pulos, J.G. and Salerno, V.L. 'Axi symmetric Elastic Deformations and stresses in a Ring Stiffened, Perfectly Circular Cylindrical Shell under External Hydrostatic pressure' David Taylor Modal Basin Report Sep 1961

[15] Salerno, V.L. and Levine, B., 'General Instability of Reinforced Shells under Hydrostatic Pressure', Report no. 189 Sep.1951.

[16] 'Stresses in Plates and Shells' A division of The MC Graw-Hill Companies-1999

[17] S. Kendrick 'The buckling shells with evenly spaced, equal strength, circular ring frames'. NCRE Report R243, Naval construction Research Establishment. 1953.

[18] S. Kendrick, 1982, 'Design for External Pressure Using General Criteria'. International Science, 1982.

[19] Timoshenko, S. 'Theory of Elasticity' MC Graw-Hill Book Company, Inc. New York, N.Y. (1936).

[20] Von Sanden, K. and Gunther, K., 'The strength of cylindrical shells, stiffened by frames and bulk heads, under uniform external pressure on all sides'. Mar. 1952.

[21] Wilson, L.B. 'The Elastic deformation of a circular cylindrical shell supported by equally spaced circular ring frames under uniform external pressure', Trans, RINA, 108, 1966.

[22] Windenburg, D.F. & Trilling, C., 'Collapse by Instability of Thin Cylindrical Shells under External Pressure, Trans, ASME, 1934.

AUTHORS PROFILE

1. P.V.MANIKANTA,
   M-TECH (MACHINE DESIGN),
   M.V.G.R. COLLEGE OF ENGINEERING,
   VIZIANAGARAM, ANDHRAPRADESH, INDIA.
   E-MAIL ID: p.manikanta876@gmail.com
   PHONE NUMBER: 91-9959932011.

2. P.RANGA RAJU, M.E
   ASSOCIATE PROFESSOR,
   MVGR COLLEGE OF ENGINEERING,
   VIZIANAGARAM, A.P, INDIA.
   E-MAIL ID: mech.mvgr@gmail.com

3. Y.APPA RAO, M.E
   SCIENTIST-E
   NSTL, VISAKHAPATNAM, A.P, INDIA.
   E-MAIL ID: ya_rao@yahoo.com