Vibration Response of Rotor-Bearing seal system subjected to Non-Linear parametric Excitations

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Abstract—

An ever-increasing pursuit of higher power and efficiency has led to highly stressed condition of rotating machine elements .To improve the performance efficiency of these kinds of machines, the radial clearance between the rotor and the stator becomes smaller and smaller. ie. operating under tighter clearances. Hence the decreasing clearance between the rotor and the stator is a necessary design. As a result, it is easier for rotor-stator rub to happen and the normal operation of machines will be affected more severely. The majority of works was focused on the development of some mathematical models in order to make the rubbing phenomenon more accurately to be understood in the past few decades. As an important component, a rotor/ ball bearing/seal assembly in the machines is a complicated multi-factor system. An accurate model of the rotor bearing assembly is essential to understand the effect of excitation

In this model many factors are taken into account, such as the rotor mass, moment inertia, bending and torsion vibration coupling effects, internal damping, the bearing contact forces and the excitation force. For determining the dynamic behaviour of the system response, Runge-Kutta method is employed to obtain the time-history diagrams, phase-plane plots; Poincare maps, whirl orbits, and frequency spectra. The experimental data obtained using Fast Fourier Transform (FFT) analyzer are validated with those obtained from numerical method. Effect of rotor speed on the periodic, quasi-periodic and chaotic motions is studied under different operating conditions.

Keywords—Ball bearing; clearance; rotor; stability; bifurcation ; chaos.

1. Introduction

In the research of rotor dynamics, the effect of bearings on rotor dynamic responses has already been taken fully into account and the rotor-bearing system dynamics has been developed [1].Many authors [2-5] worked in development of such a rotor-bearing system models by incorporating several nonlinear effects so as to get an accurate dynamic response close to the practical values. Most of the works concentrated on dynamics of rotors supported over simply supported and oil-film bearings. Even, rotors supported over rolling contact element bearings have also been utilized in literature, the external excitation forces are considered as of linear periodic type or sometimes as intermittent parametric excitations such as rub-impact forces. To the authors' knowledge, modelling of such a rotor system supported on ball bearings subjected to nonlinear excitations has not been attempted in literature.Nowadays, the dynamic responses of a fault rotor supported on oil sliding bearings have been studied extensively, however, the dynamic model of a fault rotor supported on ball bearings is still very immature, and the ball bearings' clearance and the varying compliance (VC) vibration are not considered. In the research on ball bearing vibrations, although the model of ball bearings is almost perfect, it is not combined well with the rotor vibration. Fukata et al. [6] and Mevel and Guyader [7] only considered the parameter excitation (VC vibration), which comes from varying stiffnesses, but not the effect of rotor unbalance; the unbalance force and bearing clearance were considered by Kim and Noah, but not VC vibration; the combined effect of unbalance, bearing clearance, nonlinear Hertzian contact force, and VC vibration were studied by Tiwari and Gupta [8]; but the unbalance force was considered as a constant force throughout the rotating speed range.. G.chen [9] developed a new nonlinear dynamic model of an unbalance rotor supported on ball bearings is established, in which not only the rotor unbalance, bearing clearance, nonlinear Hertzian contact force, and VC vibration are considered together, but also the unbalance force that varies with rotating speed.D.M.Ku [10] in his paper presented a finite element model belonging to C^o-class formulation for the study of whirl speeds and stability rotor-bearing systems.There has also been extensive research on issues related to rotor–stator contact-related problems. For the past two decades, majority works analyzed rub-impact analysis of rotors analytically using different mathematical models. Beatty [11] proposed a mathematical model for rubbing forces and a detailed response format of diagnostic data in actual cases. The model is still applied widely today. A comprehensive investigation on the dynamic characteristics exhibited by this kind of system is necessary in order to diagnose this fault.. Roques *et al.*[12] introduced a rotor–stator model of a turbo generator in order to investigate speed transients with rotor-to-stator rubbing caused by an accidental blade-off imbalance and highly nonlinear equations due to contact conditions are solved through an explicit prediction–correction time-marching procedure combined with the Lagrange multiplier approach dealing with a node-to-line contact strategy. and numerical. Fulei and Zhang[13] predicted the periodic ,quasi periodic, and chaotic Vibrations of a rub impact rotor system supported on oil film bearings.

Patel and Darpe [14] worked on coupled bending-torsional vibration analysis of rotor with rub and crack. In this paper, modelling and vibration signature analysis of rotor with rotor–stator rub, transverse fatigue crack and unbalance is attempted. The rotor– stator interaction effects on the response of a rotor are investigated in the presence/absence of a transverse crack. Chang-Jian and Chen [15] investigated the Chaos and bifurcation of a flexible rub-impact rotor supported by oil film bearings with nonlinear suspension The dynamic analysis of the rotor-bearing system is studied in this paper and is supported by oil film journal bearings. An observation of a nonlinearly supported model and the rub-impact between rotor and stator is needed for more precise analysis of rotor-bearing systems. Inclusive of the analysis methods of the dynamic trajectory, the power spectra, the Poincare maps, the bifurcation diagrams and the Lyapunov exponent are used to analyze the behavior of the rotor centre and bearing centre in the horizontal and vertical directions under different operating conditions. The periodic, quasi-periodic, sub-harmonic and chaotic motion are demonstrated in this study. It is concluded that the trajectory of rotor centre and bearing centre have undesirable vibrations. Appearance of the sub-synchronous frequencies in these transforms was suggested for early rub detection.

2. MATHEMATICAL MODELLING

The first model used in this work is based on a Jeffcott rotor.It is assumed that the rotor mounted on a flexible,isotrophic shaft,and simply supported by bearing at the both ends.The weight of the rotor and the shaft acts as a gravitational force which is supported by bearing force due to its stationary eccentricity.The force equilibrium of the rotor in whirling with rub impact is shown in fig 2.1.Here Os is the centre of the stator and Or is the center of the rotor.an initial cleareance of δ is provided between rotor and stator.When rubbing between rotor and stator occurs occasionally, the elastic impact must be induced . Also, friction between both contact surfaces is assumed

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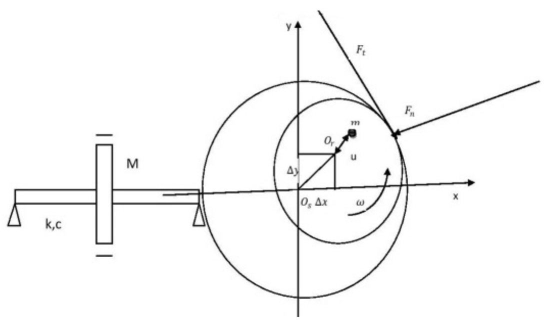

Fig 2.1:Disc-stator interaction during rub

3. RESULTS AND DISSCUSSION

3.1 Analysis of Rotor Bearing System:

Table-1 shows the parameters of shaft, disk, bearing and actuators considered in the present work for various simulation tasks.

In order to calculate natural frequency of the system first

Table-1 Data considered in present work

Shaft and Disc	Ball Bearing
$d_s = 0.015$m	N=8
$l_s = 0.400$m	r=20.0468mm
$E = 2.08\times10^{11}(N/m^2)$	R=31.953mm
$G = 0.8\times10^{11}(N/m^2)$	$\gamma_0 = 20\times10^{-6}(m)$
$\rho = 7800(kg/m^3)$	$C_b = 3.527\times10^{9}(N/m^{3/2})$
$m_d = 15.9586(kg)$	
$J'_z = 1.28\times10^{11}(kgm^2)$	
$J'_d = 0.64\times10^{11}(kgm^2)$	

linearized stiffness at the ball bearing is employed**.** The system matrices are validated by comparing natural frequencies obtained from model established in this work with results obtained by Ku [10]. Natural frequencies are calculated at 4000 rpm with shaft diameter 10.16 cm and shaft length 127 cm and stiffness and damping of damped isotropic bearing are:

$$k_{xx} = k_{yy} = 1.7513\times10^{7}(N/m), \quad k_{xy} = k_{yx} = 0$$

$$c_{xx} = c_{yy} = 1.753\times10^{3}(N-s/m), \quad c_{xy} = c_{yx} = 0$$

These stiffness values are added at in global stiffness matrix at corresponding location of ball bearing. Eigenvalues of modified system matrices are calculated using *polyeig* function of MATLAB, (which obtains eigenvalues of polynomial function).

In order to integrate differential equations of system, equations of motion are first converted into state space form (that is n second order equations are made 2n first order equations) and then solved using ode45 solver in MATLAB. For determining dynamic behavior of system response time histories, frequency spectra, whirl orbits and phase diagrams are plotted and analyzed. Fig.7(a)-(b) are time histories of horizontal vibration of rotor at disk location at various operating speeds. It can be seen from the time histories that at low speed (300 rpm) for one rotation of shaft, time response consists more than one peak which clearly indicate that motion is not periodic. But with increase in the speed of rotor amplitude of these additional peaks decreases and motion looks like periodic near first critical speed (1500 rpm).

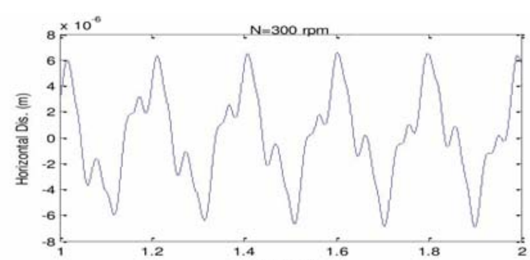

Fig. 7 (a) Time history at 300 rpm

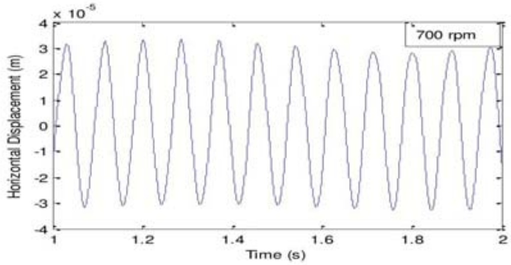

Fig. 7 (b) Time history at 700 rpm

Figures 8(a)-(b) are the corresponding frequency spectra for horizontal vibration of rotor at disk location obtained by fast Fourier transformation of time-histories with a defined sampling frequency. At low speeds frequency spectra is found to be quite dense. It consist of 1X (first harmonic) components which is dominant frequency component and higher order harmonics which are integer multiple or sum of integer multiples of VC (ball-passing) frequency and rotational frequency components.

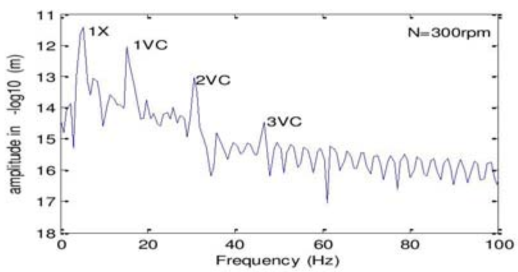

Fig. 8 (a) Frequency response at 300 rpm

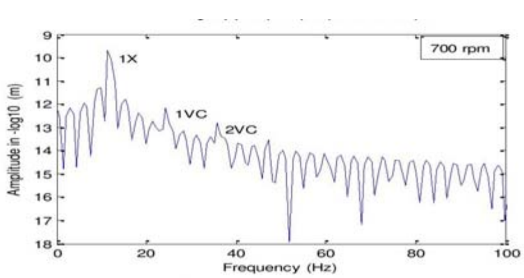

Fig. 8 (b) Frequency response at 700 rpm

At 300 rpm frequency spectra consist of 1X, 1VC, 2VC and 3VC components. At 700 rpm frequency spectra consist of 1X, 1VC and 2VC but amplitude of VC frequency component is decreased. At speeds near to first critical speed frequency spectra consists only 1X component and that is very large.

Phase plane plots

This is another tool for checking periodicity of response. Here number of loops indicates number of frequencies components in frequency response. Fig.10(a)-(b) show the phase diagrams of the rotor at disk location.

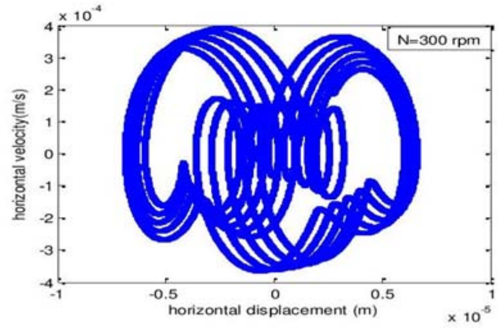

Fig. 10 (a) Phase plane plot at 300 rpm

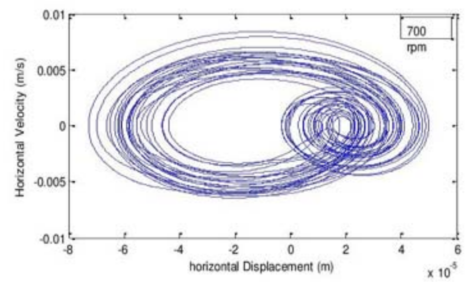

Fig. 10 (b) Phase plane plot at 700 rpm

At 300 phase plane plot consist of number of loops which indicates response is multiperiodic or quasiperiodic in nature. At 700 rpm there two loops in phase plane plot and phase trajectories of each revolution are not coinciding with each other but they are closely distributed in phase plane this indicate motion is quasi periodic with two dominant frequency components.

Poincare maps

For systems subjected to external periodic force the Poincare map is usually obtained for every time period. If the map consists of finite number of points then it indicates periodic motion and if it shows points making a curve it's a quasi periodic motion and if there are infinite number of points it shows chaotic motion. Figures 11(a)-(b) show the Poincare maps. of the rotor at disk location at various operating speeds.As there are more number of points at lower speed indicates that for every time period amplitude of motion is varying and hence the system is in chaotic state. And it is observed that as the speed is increased gradually the motion is transformed to quasi-periodic and then to periodic .

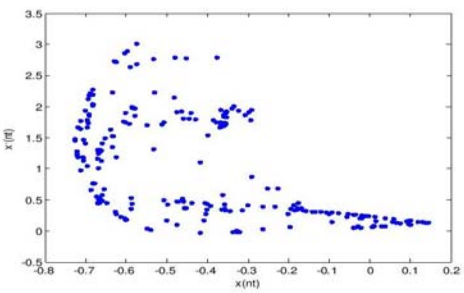

Fig .11 (a) Poincare map at 300 rpm

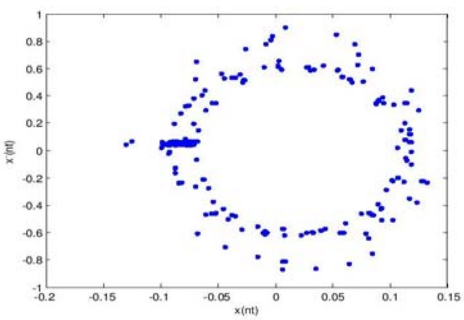

Fig .11 (b) Poincare map at 700 rpm

3.3 Effect of bearing clearance variation

Fig 13(a)-(b) show the comparison of frequency response of horizontal vibration of rotor at disk location for three different values of bearing clearance, while disk eccentricity is kept fixed at 0.1 mm. Values of clearance (in figures) are taken in meters.

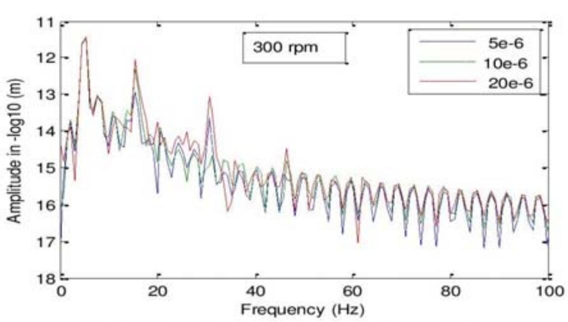

Fig. 13 (a) Frequency responses for three bearing clearance at 300 rpm

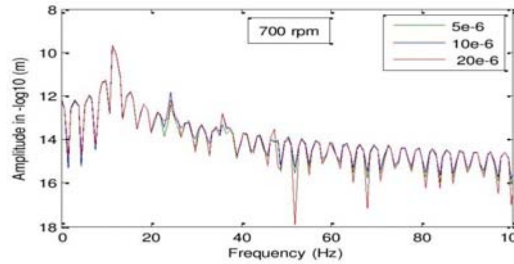

Fig. 13 (b) Frequency responses for three bearing clearance at 700 rpm

It can be seen from the figures that with the increase in bearing clearance amplitude of VC frequency component and its higher order harmonics increases while amplitude at rotational frequency remain unchanged. VC frequency components come into frequency response only at lower speeds so, effect of variation of bearing clearance is visible only at lower speeds.

3.4. Experimental Identification

The FFT analyser, which is sophisticated signal analyser of DC-12M type is dedicated to the predictive maintenance and condition diagnostics of rotating machinery by vibration as well as by the other signals converted to the electric ones. With the aid of FFT Analyser the time histories (Fig 1-4) of horizontal vibration of rotor at bearing location at various operating speeds are recorded.

It can be seen from the time histories that at speed (1080 rpm) for one rotation of shaft without consideration of rub the time history was recorded, time response consists more than one peak which clearly indicate that motion is not periodic.

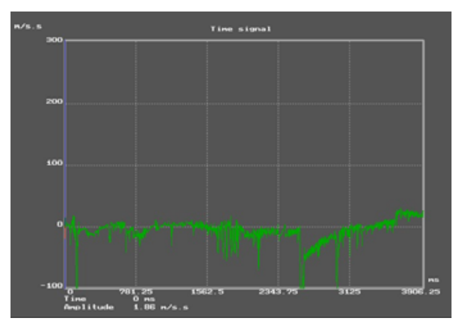

Fig.1. At a speed of 1080 rpm without rub

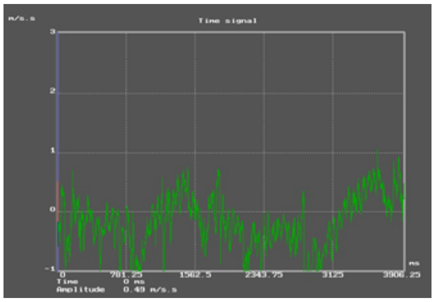

Fig.2. At a speed of 1080 rpm with rub

Fig.2 shows the time histories at a speed of 1080 rpm with the consideration of rub at 20 degrees from the centre of the disk.

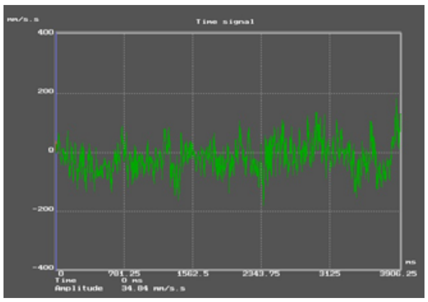

Fig.3. At a speed of 1280 rpm with rub

Fig.3 shows the time history at a speed of 1280 rpm with the consideration of rub at 20 degrees from the centre of the disk.

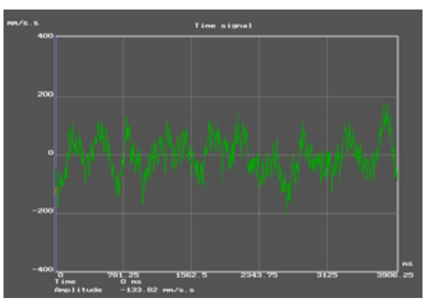

Fig.4. At a speed of 1380 rpm with rub

Fig.4 shows the time history at a speed of 1380 rpm with the consideration of rub at 20 degrees from the centre of the disk.

Time history plots are very uneven in case of lower speeds and the effect of variation of bearing clearance is visible only at lower speeds. But with increase in the speed of rotor amplitude of these additional peaks decreases and motion looks like periodic near the first critical speed.

4.CONCLUSION

From the present study of dynamic response of an unbalanced flexible rotor supported on ball bearings (with and without rub), Effect of parametric excitation of ball bearing on system response is significant only at low speed, around first natural frequency. Increase in disk unbalance increases amplitude of rotational frequency component but it does not affect amplitude of variable frequency component. Rub influence is very much higher in lower speed than at elevated speeds. The experiment is to be conducted very carefully at different speeds and with different rub angles so that there may be a chance of getting very important insights in the rotor dynamic research, this is an attempt made to get the insights of the rotor dynamic system.

5.REFERENCES

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