New Optimal Solution to Fuzzy Interval Transportation Problem

B. Ramesh Kumar S. Murugesan

Department of Mathematics, Sree Sowdambika College of Engineering, Aruppukotai,

Department of Mathematics, S.R.N.M College of Arts and Science,Sattur, Tamil Nadu,

ABSTRACT - In this paper, Fuzzy interval method is proposed to find an optimal solution for a fuzzy transportation problem. The Transportation costs, supply and demand values are considered in an fuzzy intervals. The solution procedure is illustrated with a numerical example. The fuzzy interval method can serve as an important tool for the decision making.

Keywords: Fuzzy triangular number; Fuzzy transportation problem; Fuzzy interval transportation problem; Zero termination method

1. INTRODUCTION

Transportation problem is a special class of linear programming problem which deals with the distribution of a single commodity from various sources to different destinations of demand, in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be tried at crisp values. But in real life application supply, demand and unit transportation cost may be uncertain due to several factors. These imprecise data may be represented by fuzzy numbers. The idea of fuzzy set was introduced by Zadeh [12] in 1965. Bellmann and Zadeh [2] proposed the concept of decision making in fuzzy environment. After this many authors have suggested various efficient methods for solving transportation problems with the assumption of precise source, destination parameters and the penalty factors. In real life problems, these assumptions may not be always possible. To deal with inexact coefficients in transportation problems many researchers [3, 6, 9, 5] have proposed fuzzy and interval programming techniques for solving the minimal transportation cost.

Des et al [4], proposed a method, using fuzzy technique to solve interval transportation problems by considering the right bound and the midpoint interval. Pal [10] proposed a new orientated method to solve interval transportation problems by considering the midpoint and width of the interval in the objective function. S.Mohanaseslvi and K.Ganesan [8] proposed the initial fuzzy feasible solution to a fuzzy transportation problem. But most of the existing techniques provide only crisp solution for fuzzy transportation problem. The authors obtained only crisp optimal solution to a given fuzzy transportation problem. In this paper, we propose a new algorithm to find the initial fuzzy feasible solution to a fuzzy transportation problem without converting to a classical transportation problem.

The rest of the paper is organized as follows. In section 2, we recall the basic concepts and results of triangular fuzzy numbers and the fuzzy transportation problem with triangular fuzzy number and their related results. In Section 3, we establish the optimality of the solution obtained using the new algorithm. In Section 4, we summarize the new algorithm is given. In Section 5, numerical example is illustrated.

2. PRELIMINARIES

2.1. Definition [2] A fuzzy set A defined on the set of real number R is said to be a fuzzy number if its membership function $A: R \rightarrow [0\ 1]$ has the following characteristics;

(i) A is convex(ie)

$$A\{\lambda x_1 + (1-\lambda)x_2\} = \max\{x_1,\ x_2\}\ forall\ \ x_1, x_2 \in R$$

(ii) $\bar{a}$ is normal (ie) $\exists\ an\ \ x \in R\ suchthat\ \ A(x) = 1$

(iii) $\bar{a}$ is piecewise continuous

2.2. Definition [9] A fuzzy number A on R is said to be a triangular fuzzy number (TFN) if its membership

$$\mu_A(x) = \begin{cases} 0 & for\ \ x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & for\ \ a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & for\ a_2 \le x \le a_3 \\ 0 & for\ \ x > a_3 \end{cases}$$

function $A: R \rightarrow [0\,1]$ has the following form

It is denoted this triangular fuzzy number $A = (a_1\, a_2\, , a_3)$.

F(R) to denote the set of all triangular fuzzy numbers.

2.3. Definition [7] Ming Ma et al, have proposed a new fuzzy arithmetic operation on fuzzy interval number based upon both location index and fuzziness index functions. For arbitrary triangular fuzzy numbers $A=(a_1\,a_2\,,a_3)$ and $B=(b_1\,b_2\,,b_3)$, defined as the arithmetic operation is

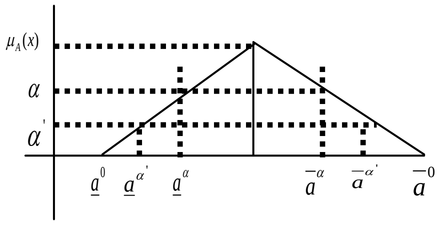
Multiplication: $A*B=(\ \max(a_1\,b_1)\ ,\ a_2b_2\ ,\ \max(a_3b_3)\ )$

Addition : $A+B=(\ \max(a_1\,b_1)\ ,\ a_2+b_2\ ,\ \max(a_3b_3)\ )$

Subtraction: $A-B=(\ \max(a_1\,b_1)\ ,\ a_2-b_2\ ,\ \max(a_3b_3)\ )$

2.4 α -cut operation on fuzzy numbers [11]

Generally a fuzzy interval is represented by two points $\underline{a},\overline{a}$ be the peak points of 'A ' as $[\underline{a},a,\overline{a}]$. The α -cut operation can also be applied to a fuzzy number. If we denote the α -cut interval for the fuzzy number as A_α, the obtained interval A_α is defined as $A_\alpha=[\underline{a}^\alpha,\overline{a}^\alpha]$.



It is observed that $\alpha'<\alpha\Rightarrow a_{\alpha'}<a_\alpha$: Indeed, $\underline{a}^{\alpha'}\le\underline{a}^\alpha$, $\overline{a}^{\alpha'}\le\overline{a}^\alpha$.

2.4 (a) Operation on fuzzy numbers can be generalized from that of crisp interval as follows: $\forall\ a,b,\underline{a},\underline{b},\overline{a},\overline{b}\in R$, if $A=[\underline{a},a,\overline{a}]$ and $B=[\underline{b},b,\overline{b}]$

then
$$A+B=\left\{\ \underline{a}+\underline{b}\ ,\ a+b,\ \overline{a}+\overline{b}\ \right\}$$
$$A-B=\left\{\ \underline{a}-\overline{b}\ ,\ a-b,\ \overline{a}-\underline{b}\right\}$$

2.4 (b) Fuzzy Interval α cuts are

(i) $A_\alpha=[\underline{a}^\alpha,\overline{a}^\alpha]=[(a-\underline{a})\alpha+\underline{a}\ ,\ (a-\overline{a})\alpha+\overline{a}\ ]$

(ii) $B_\alpha=[\underline{b}^\alpha,\overline{b}^\alpha]=[(b-\underline{b})\alpha+\underline{b}\ ,\ (b-\overline{b})\alpha+\overline{b}\ ]$

(iii) Product of α -cut $R_\alpha=A_\alpha\times B_\alpha$ =

$[\underline{a}^\alpha,\overline{a}^\alpha]$ x $[\underline{b}^\alpha,\overline{b}^\alpha]$ = $[\ \underline{R}^\alpha\ ,\overline{R}^\alpha]$ Where

$$\underline{R}^\alpha=\min(\underline{a}^\alpha\underline{b}^\alpha,\underline{a}^\alpha\overline{b}^\alpha,\overline{a}^\alpha\underline{b}^\alpha,\overline{a}^\alpha\overline{b}^\alpha)\quad and$$

$$\overline{R}^\alpha=\max(\underline{a}^\alpha\underline{b}^\alpha,\underline{a}^\alpha\overline{b}^\alpha,\overline{a}^\alpha\underline{b}^\alpha,\overline{a}^\alpha\overline{b}^\alpha)$$

2.5. Definition [1] If $A=[\underline{a},a,\overline{a}]$ is a Triangular Fuzzy Number, then the measure is defined as $M^{TRI}(A)=\frac{1}{4}(2a+\underline{a}+\overline{a})$. It satisfies $A\le B\ iff\ M(A)\le M(B)$

3 Formation of Fuzzy Transportation Problem and solutions:

Consider fuzzy transportation problem with m sources and n destinations. Let a_i be the fuzzy availability at source i and b_j be the fuzzy requirements at destination j. Let c_{ij} be the fuzzy unit transportation cost from source i to destination j. Let x_{ij} denote the number of fuzzy units to be transported from source i to destination j. Then the problem is to determine a feasible way of transporting the available quantity at each source to satisfy the demand at each destination, so that the total transportation cost is minimum. The mathematical formulation of the fuzzy transportation problem whose parameters are fuzzy numbers under the given constraint is;

$$\min z=\sum_i\sum_j c_{ij}x_{ij}$$

$$\text{s.t}\sum_{j=1}^{n}x_{ij}=a_i\quad i=1,2..m$$

$$\sum_{i=1}^{m}x_{ij}=b_j\quad j=1,2..n$$

$$\sum_{i=1}^{m}a_i=\sum_{j=1}^{n}b_j\quad and\quad x_{ij}\ge 0$$

3.1 (a) Fuzzy basic feasible solution: A fuzzy feasible solution to a fuzzy transportation problem with m sources and n destinations is said to be a fuzzy basic feasible solution if the number of positive allocations (m+n-1).

3.1 (b) If the number of positive allocations in a fuzzy solution is less than (m+n-1), it is called a fuzzy degenerate feasible solution.

3.1(c) Fuzzy optimal costs: A fuzzy feasible solution is said to be a fuzzy optimal solution if it minimizes the total fuzzy transportation cost.

3. 2 FUZZY INTERVAL TRANSPORTATION PROBLEM;

If the non-negative fuzzy quantities $a_{i,}\ b_j,x_{ij},c_{ij}$ in a fuzzy transportation problem are taken as fuzzy intervals then the fuzzy transportation problem becomes a fuzzy interval transportation problem.

3.2 (a) Definition;

A feasible solution $[\boldsymbol{u_{ij}}\ \boldsymbol{v_{ij}}\ \boldsymbol{w_{ij}}]$ for all i = 1, 2...m, j = 1, 2...n of the FITP is said to be an optimal solution. If $\sum_i\sum_j[\underline{c_{ij}}\ c_{ij}\ \overline{c_{ij}}]\otimes[\underline{x_{ij}}\ x_{iij}\ \overline{x_{ij}}]\ \sum_i\sum_j[\underline{\boldsymbol{c_{ij}}}\ \boldsymbol{c_{ij}}\ \overline{\boldsymbol{c_{ij}}}]\otimes[\boldsymbol{u_{ij}}\ \boldsymbol{v_{ij}}\,\boldsymbol{w_{ij}}]$

for i = 1,2..m, j = 1,2..n and for all feasible $[u_{ij}\ v_{ij}\ w_{ij}]$. Now, we prove the following theorem which finds a relation between optimal solutions of a fuzzy transportation problem and a pair of induced transportation problems. It is used in this proposed method.

3.2.1 Proposition;

Let the set $\overline{x_{ij}}$ be an optimal solution of upper bound fuzzy interval transportation,

$$\min \overline{z_{ij}} = \sum_i \sum_j \overline{c_{ij}}\, z_{ij}$$

where $$s.t \sum_j \overline{x_{ij}} = \overline{a_i}, \quad i = 1,2..m \qquad \text{-------- (1)}$$

$$\sum_i \overline{x_{ij}} = \overline{b_j}, \quad j = 1,2...n$$

if the set $\underline{x_{ij}}$ be an optimal solution of the lower bound fuzzy interval transportation,

$$\min \underline{z_{ij}} = \sum_i \sum_j \underline{c_{ij}}\, z_{ij}$$

where $$s.t \sum_j \underline{x_{ij}} = \underline{a_i}, \quad i = 1,2..m \qquad \text{----------------- (2)}$$

$$\sum_i \underline{x_{ij}} = \underline{b_j}, \quad j = 1,2...n$$

If the set x_{ij} is an optimal solution of the fuzzy transportation problem,

$$\min z_{ij} = \sum_i \sum_j c_{ij}\, z_{ij}$$

where $$s.t \sum_j x_{ij} = a_i, \quad i = 1,2..m \quad , \text{--------- (3)}$$

$$\sum_i x_{ij} = b_j, \quad j = 1,2...n$$

then the set of intervals $[\underline{x_{ij}}, x_{ij}, \overline{x_{ij}}]$ is an optimal solution of the FITP provides $\underline{x_{ij}} \le x_{ij} \le \overline{x_{ij}}$

Proof;

Let $[\boldsymbol{u_{ij}}\ \boldsymbol{v_{ij}}\ \boldsymbol{w_{ij}}]$ & $[\boldsymbol{x_{ij}}\ \boldsymbol{y_{ij}}\ \boldsymbol{z_{ij}}]$ be a fuzzy feasible solution of the FITP

Where $[\boldsymbol{x_{ij}}\ \boldsymbol{y_{ij}}\ \boldsymbol{z_{ij}}]$ be the fuzzy triangular number, $\boldsymbol{x_{ij}}$, $\boldsymbol{z_{ij}}$ be the lower bound FITP and upper bound FITP

Now, since $[\underline{x_{ij}}, x_{ij}, \overline{x_{ij}}]$ are optimal solutions of the FITP, we have

$$\sum_i \sum_j \overline{c_{ij}}\, \overline{z_{ij}} \le \sum_i \sum_j \overline{c_{ij}}\, w_{ij},,$$

$$\sum_i \sum_j c_{ij}\, x_{ij} \le \sum_i \sum_j c_{ij}\, y_{ij},, \quad \text{and}$$

$$\sum_i \sum_j \underline{c_{ij}}\, \underline{x_{ij}} \le \sum_i \sum_j \underline{c_{ij}}\, x_{ij},, \quad \textbf{and } \boldsymbol{u_{ij} \le v_{ij} \le w_{ij}}$$

$$\left[\sum_i \sum_j \underline{c_{ij}}\,\underline{x_{ij}}, \sum_i \sum_j c_{ij} x_{ij}, \sum_i \sum_j \overline{c_{ij}}\,\overline{x_{ij}}\right] \le$$

$$\left[\sum_i \sum_j \underline{c_{ij}} u_{ij}, \sum_i \sum_j c_{ij} v_{ij}, \sum_i \sum_j \overline{c_{ij}} w_{ij}\right]$$

Precisely,

$$\sum_i \sum_j \left[\underline{c_{ij}}, c_{ij}, \overline{c_{ij}}\right] \otimes \left[\underline{x_{ij}}, x_{ij}, \overline{x_{ij}}\right] \le$$

$$\sum_i \sum_j \left[\underline{c_{ij}}, c_{ij}, \overline{c_{ij}}\right] \otimes \left[u_{ij}, v_{ij}, w_{ij}\right]$$

Now, since $[\underline{x_{ij}}, x_{ij}, \overline{x_{ij}}]$ satisfies the min $[\underline{z_{ij}}\ z_{ij}\ \overline{z_{ij}}]$ for all i and j.

we can conclude that the set $[\underline{x_{ij}}, x_{ij}, \overline{x_{ij}}]$ is a feasible solution of the problem.

Thus the set of intervals $[\underline{x_{ij}}, x_{ij}, \overline{x_{ij}}]$ is an optimal solution of the FITP.

4. ZERO TERMINATION METHOD (Z_T)

The procedure of Zero termination method is as follows,

Step 1: Construct the transportation table

Step 2: select the smallest unit transportation cost value for each row and subtract it from all costs in that row. In a similar way this process is repeated column wise.

Step 3: In the reduced cost matrix obtained from step 2, there will be at least one zero in each row and column. Then we find the termination value of all the zeros in the reduced cost matrix, using the following rule; the zero termination cost is T= Sum of the costs of all the cells adjacent to zero is divided by the Number of non- zero cells added in the sum

Step 4: In the revised cost matrix with zero termination costs if it has a unique maximum T, allocate maximum possible to that the cell. If it has more one, then the select the cell with the largest cost and allocate the maximum possible.

Step 5: After the allocating the columns and rows corresponding to exhausted demands and supplies are trimmed. The resultant matrix must possess at least one zero in each row and column, else repeat step (2)

Step 6: Repeat step (3) to step (5) until the optimal solution is obtained.

5. EXAMPLE

Consider the following fuzzy interval transportation problem of minimal cost representation; the optimal cost is obtained by the fuzzy intervals using a new algorithm. The problem is a balanced fuzzy transportation problem.

Table 1: Fuzzy Transportation problems

					Capacity
	(-2 3 8)	(-2 3 8)	(-2 3 8)	(-11 4)	(0 3 6)
	(4 9 16)	(4 8 12)	(2 5 8)	(1 4 7)	(2 7 13)
	(2 7 13)	(0 5 10)	(0 5 10)	(4 8 12)	(2 5 8)
Demand	(1 4 7)	(0 3 5)	(1 4 7)	(2 4 8)	

Using the relation (2.7), the given fuzzy transportation problem is converted into a fuzzy interval transportation problem. Then, the optimal solution to the fuzzy interval transportation problem is obtained by using our proposed method, the initial fuzzy transportation cost is obtained from (Table **2)**

					capacity
	$(2+5\alpha,3,3-5\alpha)$ $(3\alpha,3,\ 6-3\alpha)$	$(2+5\alpha,3,3-5\alpha)$	$(2+5\alpha,3,3-5\alpha)$	$(-1+2\alpha,1,4-3$	$(3\alpha,3,6-3\alpha)$
	$(4+5\alpha,9,16-7\alpha)$ $(1+0\alpha,1,1-0\alpha)$	$(4+4\alpha,8,12-4\alpha)$	$(2+3\alpha,5,8-3\alpha)$ $(-1+3\alpha,2,4-2\alpha$	$(1+3\alpha,4,7-3\alpha)$ $(2+2\alpha,4,8-4\alpha$	$(2+5\alpha,7,13-6$
	$(2+5\alpha,7,13-6\alpha)$	$(5\alpha,5,10-5\alpha)$ $(3\alpha,3,5+2\alpha)$	$(5\alpha,\ 5,\ 10-5\alpha)$ $(2+0\alpha,2,3-5\alpha)$	$(4+4\alpha,8,12-4\alpha$	$(2+3\alpha,5,8-3\alpha)$
Demand	$(1+3\alpha,4,7-3\alpha)$	$(3\alpha,3,5+2\alpha)$	$(1+3\alpha,4,7-3\alpha)$	$(2+2\alpha,4,8-4$	

It can be seen that the current initial fuzzy basic feasible solution is optimum.

$$= (2+5\alpha,3,3-3\alpha)*(3\alpha,3,6-3\alpha) + (4+5\alpha,9,16-7\alpha)*(1+0\alpha,1,1-0\alpha) + (2+3\alpha,5,8-3\alpha)*(-1+3\alpha,2,4-2\alpha) + (1+3\alpha,4,7-3\alpha)*(2+2\alpha,4,8-4\alpha) + (5\alpha,5,10-5\alpha)*(3\alpha,3,5+3\alpha) + (5\alpha,5,10-5\alpha)*(2+0\alpha,2,3-5\alpha)$$

$$= (2+5\alpha,9,6-3\alpha) + (4+5\alpha,9,16-7\alpha) + (2+3\alpha,10,8-3\alpha) + (2+2\alpha,16,8-4\alpha) + (5\alpha,15,10-5\alpha) + (4+4\alpha,16,12-4\alpha)$$

$$= (4+5\alpha,75,12-4\alpha)$$

Hence the fuzzy optimal solution in terms of location index an fuzziness index is given by

$x_{11}=(3\alpha,3,6-4\alpha)$, $x_{21}=(1+0\alpha,1,1-0\alpha)$, $x_{23}=(-1+3\alpha,2,4+2\alpha)$, $x_{24}=(2+2\alpha,4,8-4\alpha)$, $x_{32}=(3\alpha,3,5+2\alpha)$, $x_{33}=(2+0\alpha,2,3-5\alpha)$

When α =0, we get

$x_{11}=(0,3,6)$, $x_{21}=(1,1,1)$, $x_{23}=(-1,2,4)$, $x_{24}=(2,4,8)$, $x_{32}=(0,3,5)$, $x_{33}=(2,2,3)$.

The corresponding fuzzy optimal transportation cost is given by

= (2,3,8)(0,3,6) + (4,9,26)(1,1,1) + (2, 5, 8)(-1, 2, 4)+ (1, 4, 7) (2, 4, 8) + (0, 5, 10)(3, 3, 5) + (0, 5, 10) (2, 2, 3)

= (0, 9, 8) + (4, 9, 26) + (2, 10, 8) + (2, 16, 8) + (3, 15, 10) + (2, 10, 10)

= [13, 69, 70] = 55.25.

The fuzzy optimal solution is obtained by the problem [10] using $x_{11}=(3,3,3)$, $x_{23}=(3,5,6)$, $x_{24}=(4,2,4)$, $x_{31}=(1,5,6)$, $x_{32}=(3,5,2)$, $x_{33}=(1,5,6)$ and also, the minimum fuzzy transportation cost as RS[62, 67,73] = 67.25.

6. CONCLUSION

In this paper the new algorithm is proposed to fuzzy optimal solution to a fuzzy interval transportation problem using zero termination method, next the triangular fuzzy number represents the fuzzy interval transportation problem in terms of the upper bound and lower bound index function of fuzzy intervals, and it is tried by the proposed new algorithm. Finally, we infer that the proposed method provides the better optimum to fuzzy interval transportation problem and it can serve an important tool in decision making problem.

REFERENCES

[1] H.Basirzadeh, An approach for solving fuzzy transportation problem, Appl.math.Sci.5 (2011) 1549-1566

[2] B.E.Bellmann and L.A.Zadeh. Decision making in fuzzy environment management sciences, 17(1970), 141-164

[3] S.Chanas and D.Kuchta, A concept of solution of the transportation problem with fuzzy coscoefficients, fuzzy sets and systems, 82(1996), 200-305

[4] S.K.das. A.Goswami and S.S.Alam, multiobjective transportation problem with interval cost, source and destination parameters, EJOR, 117(1999).100-112

[5] F.Herrera and J.L.Verdegay, three model of fuzzy integer linear programming, EJOR 83(1995) 581-593

[6] H.Ishibuchi and H.Tanka, multiobjective programming in optimization of the interval objective function, EJOR 48(1990).219-225

[7] Ming Ma, Menahem Friedman, Abraham Kandel, A new fuzzy arithmetic. Fuzzy sets and systems, 108, (1999) ,83-90

[8] S.Mohanaselvi and K.Ganesan, A new approach fuzzy optimal solution to fuzzy transportation problem, IJCSE, Vol 4.No.3 (2012), 367-375

[9] P. Pandian and G.Natarajan, A new algorithm for finding a fuzzy optimal solution for FTP App.math.Sci, 4(2010), 79-90

[10] A.Sengubta and T.K.Pal, Interval valued transportation problem with multiple penalty factors, Jour.of.phy.sci. 9(2003), 71-81

[11] ShanGao and ZaiyueZhang, multiplication on fuzzy numbers, Jour.of.software, Vol.4,No.4(2009), 331-337

[12] L.A.Zadeh, Fuzzy sets, information and control, 8 (1965), 338-353,