APPROXIMATE SOLUTION OF TWO PHASE THERMAL BOUNDARY LAYER FLOW

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Abstract **— The laminar, incompressible viscous two-phase thermal boundary layer flow over a flat plate is studied. The momentum integral method has been employed to study the boundary layer characteristics. The particle velocity, density and temperature approaches a finite value towards the downstream of the plate, and the solution is valid throughout the plate, which is a better result than the solution available in the literature. It has been observed that, heat flows from the plate towards the fluid as Nusselt number (Nu) is positive. The numerical value of particle temperature with frictional heat is less than that of without frictional heat. Further, the numerical value of Nusselt number with frictional heat is greater than that of without frictional heat, indicating that inclusion of frictional heat increases the heat transfer from plate to fluid.**

Keywords- Two-phase flow; Boundary layer characteristics; Shear Stress; Heat transfer

Nomenclature

$(x, y,)$	Space co-ordinates i.e. distance along the perpendicular to plate length
$\vec{q}(u, v)$	Velocity components for the fluid phase in x and y− directions respectively
$\vec{q}_p(u_p, v_p)$	Velocity components for the particle phase in x and $y-$ directions respectively
(T, T_p)	Temperature of fluid and particle phase respectively
(T_w, T_∞)	Temperature at the wall and free-stream respectively
(υ, υ_p)	Kinematic coefficient of viscosity of fluid and particle phase respectively
(ρ, ρ_p)	Density of fluid and particle phase respectively
ρ_s	Material density of particle (Rhop)
(μ, μ_p)	Coefficient of viscosity of fluid and particle phase respectively
(τ_p, τ_T)	Velocity and thermal equilibrium time
(c_p, c_s)	Specific heat of fluid and SPM respectively
Re	Fluid phase Reynolds number
Pr	Prandtl number
Ec	Eckret number
Nu	Nusselt number
c_f	Skin friction coefficient
τ_w	Skin friction (Shear stress for clear fluid)
p	Pressure of fluid phase
φ	Volume fraction of Suspended particulate matter (SPM)
D	Diameter of the particle
δ	Boundary layer thickness
a	Thermal diffusivity
κ	Thermal conductivity
α	Concentration parameter(Alpha)
ϵ	Diffusion parameter
F	Friction parameter between the fluid and the particle $(F = 18\mu/\rho_p d^2)$
L	Reference / Characteristic length
U	Free stream velocity
A	δ^2/L^2
SPM	*S*uspended particulate matter
Subscripts	
0 or ∞	Free Stream value
w	Plate value

I. INTRODUCTION

It is of great importance to know the structure of the two-phase boundary layer flow and heat transfer over a flat plate and to estimate the surface characteristics like skin friction co-efficient, wall heat transfer rate, particulate velocity and density on the surface under various assumptions. Several investigators [1-9] have derived equations governing the Two Phase flow and reduce them to boundary layer type using Prandtl boundary layer approximations. Marbel's [2] solution is valid for downstream region of the plate and the particulate velocity on the surface remains zero. Singleton [6] has studied compressible gas particulate flow over a flat plate for high and low slip flow regions by employing series solution method. Soo [7] has derived momentum integrals for the gas and particulate phases and solved the same by using linear profiles both for gas phase and particle phase and quadratic profile for particulate density. Tabakoff and Hammed [9] have used fourth degree profiles for both gas and particle velocity and particle density. Soo [7] and Tabakoff and Hammed [9] have pointed out that particle velocity decreases linearly along the plate length x and particle density increases continuously along the plate length x. Their study leads to a surface particle velocity zero and particle density to infinity at a distance along the plate length $x = 1$. No effort has been made for studying the temperature distribution inside the boundary layer. Jain & Ghosh [1] have investigated the structure and surface property of the boundary layer of a gas particulate flow over a flat plate by employing momentum integral method. They have pointed out that the third degree profile for velocity and particle density gives results which is valid to far downstream stations on the plate. With the third degree profile of particulate velocity on the surface continuously decreases from its free stream value and particulate density on surface increases rather slowly from its free stream value at the leading edge to an asymptotic value as we approach far downstream on the plate surface. Mishra & Tripathy[4] have investigated the two-phase boundary layer flow over a flat plate to study the boundary layer flow characteristics without considering volume fraction, viscous terms in particle momentum equation and conduction of heat in particle phase energy equation.

The present study is an attempt to study the temperature distribution inside the boundary layer over a flat plate, which gives a better understanding of the gas particulate boundary layer flow. In this case, the momentum integral method is adopted to study the flow and temperature distribution by using a third degree profiles.

II. MATHEMATICAL FORMULATION & SOLUTION

The governing equations of steady two dimensional gas-particulate flow within the boundary layer on a flat plate, are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial}{\partial x}\left(\varphi\rho_S u_p\right) + \frac{\partial}{\partial y}\left(\varphi\rho_S v_p\right) = 0 \tag{2}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\varphi}{1-\varphi}\frac{\rho_s}{\tau_p}\left(u - u_p\right) \tag{3}$$

$$\varphi\rho_S\left(u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}\right) = \frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right) + \varphi\frac{\rho_s}{\tau_p}\left(u - u_p\right) \tag{4}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\varphi}{1-\varphi}\frac{\rho_s c_s}{\tau_T}\left(T_p - T\right) \tag{5}$$

$$\varphi\rho_s c_s\left(u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y}\right) = \frac{\partial}{\partial y}\left(\varphi\kappa_s\frac{\partial T_p}{\partial y}\right) + \varphi\rho_s c_s\frac{1}{\tau_T}\left(T - T_p\right) \tag{6}$$

Subject to the boundary conditions

At $y = 0 : u = 0,\ v = 0,\ u_p = a_2(x),\ v_p = 0,\ \rho_p = a_3(x),$

$$T = T_w,\ \ T_p = a_4(x) \tag{7}$$

At $y = \delta :\ u = U,\ u_p = U,\ \rho_p = \rho_{p_\infty},\ T = T_\infty,\ T_p = T_\infty$ (8)

Considering the carrier fluid as incompressible, μ and κ are constant and if the temperature variation is small; μ_s and k_s may be taken as constant. Here the term $\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right)$ may be replaced by $\varphi\mu_s\frac{\partial^2 u_p}{\partial y^2}$, in the particle phase x-momentum equation which arises due to the particle random motion in direct correspondence with similar terms for fluid phase and the term $\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)$ in energy equation for particle phase may be replaced by $\varphi k_s\frac{\partial^2 T_p}{\partial y^2}$. As the free stream velocity U is independent of x, $\frac{\partial p}{\partial x} = 0$.

Clearly $\delta > \delta_t$ and $\delta > \delta_p$

It may be noted that, the thickness of the thermal boundary layer (δ_t), particle velocity boundary layer (δ_p), particle thermal boundary layer (δ_{p_t}) are the same as that of the velocity boundary layer(δ). Strictly speaking, they are different, in general. This assumption has its justification in that it simplifies the computational work and the results obtained are very near to the experimental results and to the exact solutions.

Now, on integration equations (3) to (6) w. r. t. y from $y = 0$ (wall) to $y = \delta$, we get

$$\frac{\partial}{\partial x}\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \frac{\mu}{\rho U^2}\left.\frac{\partial u}{\partial y}\right|_0 + \frac{1}{1-\varphi}F\int_0^\delta \frac{\rho_p}{\rho U}\left(1 - \frac{u_p}{U}\right)dy - \frac{1}{1-\varphi}F\int_0^\delta \frac{\rho_p}{\rho U}\left(1 - \frac{u}{U}\right)dy \tag{9}$$

$$\frac{\partial}{\partial x}\int_0^\delta\left(\rho_p u_p\right)\left(U - u_p\right)dy = \varphi\,\mu_s\left.\frac{\partial u_p}{\partial y}\right|_{y=0} - F\int_0^\delta \rho_p\left(u - u_p\right)dy \tag{10}$$

$$\frac{\partial}{\partial x}\int_0^\delta u\left(T - T_\infty\right)dy = -a\left.\frac{\partial T}{\partial y}\right|_{y=0} + \frac{\mu}{\rho c_p}\int_0^\delta\left(\frac{\partial u}{\partial y}\right)^2 dy + \frac{1}{1-\varphi}\frac{1}{\tau_T}\frac{c_s}{\rho c_p}\int_0^\delta \rho_p\left(T_p - T\right)dy \tag{11}$$

$$-\frac{\partial}{\partial x}\int_0^\delta\left(\rho_p u_p\right)\left(T_p - T_\infty\right)dy = \frac{\varphi\,k_p}{c_s}\left.\frac{\partial T_p}{\partial y}\right|_{y=0} + \frac{1}{\tau_T}\int_0^\delta \rho_p\left(T_p - T\right)dy \tag{12}$$

By introducing the non- dimensional quantities

$x^* = \frac{x}{L},\ y^* = \frac{y}{\delta},\ u^* = \frac{u}{U},\ u_p^* = \frac{u_p}{U},\ \rho_p^* = \frac{\rho_p}{\rho_{p_0}}$,

$$T^* = \frac{T - T_\infty}{T_w - T_\infty}\ ,\quad T_p{}^* = \frac{T_p - T_\infty}{T_{pw} - T_\infty} \tag{13}$$

in the equations (9) to (12) and after dropping stars, we get

$$\frac{\partial}{\partial x}\left[\frac{\delta}{L}\int_0^1 u(1-u)\,dy\right] = \frac{\mu}{\rho U\delta}\left.\frac{\partial u}{\partial u}\right|_{u=0} + \frac{1}{1-\varphi}\,F\,\frac{\rho_{p_0}}{\rho}\frac{\delta}{U}\int_0^1 \rho_p\left(1-u_p\right)dy - \frac{1}{1-\varphi}\,F\,\frac{\rho_{p_0}}{\rho}\frac{\delta}{U}\int_0^1 \rho_p\,(1-u)\,dy \tag{14}$$

$$\frac{\partial}{\partial x}\delta\int_0^1 \left(\rho_p\,u_p\right)\left(1-\ u_p\right)\,dy = \frac{L^2}{\delta}\frac{\epsilon}{Re}\left.\frac{\partial u_p}{\partial y}\right|_{y\,=\,0} - \delta\frac{FL}{U}\int_0^1 \rho_p\left(u-u_p\right)dy \tag{15}$$

$$\frac{\partial}{\partial x}\left[\delta\ \int_0^1 u\,T\,dy\right] = -\frac{\alpha L}{U\delta}\left.\frac{\partial T}{\partial y}\right|_{y=0} + \frac{\mu}{\rho c_p}\frac{UL}{\delta(T_w - T_\infty)}\int_0^1\left(\frac{\partial u}{\partial y}\right)^2 dy - \frac{1}{1-\varphi}\frac{c_s}{c_p}\ \frac{\rho_{p_0}}{\rho}\left(\frac{\partial}{\partial x}\left\{\delta\int_0^1 \rho_p\,u_p\,T_p dy\right\} - \frac{L^2}{\delta}\ \frac{\epsilon}{\Pr Re}\left.\frac{\partial T_p}{\partial y}\right|_{y=0}\right) \tag{16}$$

and the boundary conditions (7) & (8) reduces to

$$y = 0\ :\ u = 0,\ v = 0,\ u_p = a_2(x),\ v_p = 0,\ \rho_p = a_3(x),\ T = 1,\ T_p = a_4(x) \tag{17}$$

$$y = 1\ :\ u = u_p = \rho_p = 1,\ T = 0,\quad T_p = 0 \tag{18}$$

For consistency, we use the auxiliary condition that the flux of particulate mass across any control volume is zero.

i.e. $$\rho_{p0}U\delta = \int_0^\delta \rho_p u_p\ dy \tag{19}$$

which gives in non – dimensional form as

$$\frac{d}{dx}\int_0^1 \rho_p\,u_p\ dy = 0 \tag{20}$$

Using the following profiles [1] satisfying the boundary conditions (17, 18) as these profiles gives results to far-downstream station on the plate,

$$\begin{aligned} u &= 1-(1-\ y)^3 \\ u_p &= 1-\ (1-a_2)\,(1-\ y)^3 \\ \rho_p &= 1-\ (1-a_3)\,(1-\ y)^3 \\ T &= (1-\ y)^3 \\ T_p &= a_4(1-\ y)^3 \end{aligned} \tag{21}$$

in the equations (14) to (16), and by suppressing the term due to the frictional heat (2nd term of R.H.S. in eqn. (16)) , we get

$$\frac{dA}{dx} = \frac{56\mu}{\rho UL} - \frac{2}{3}\ \frac{FL}{U}\alpha A a_2(4a_3 + 3) \tag{22}$$

$$\frac{da_2}{dx} = \frac{\left\{\begin{array}{c}\frac{dA}{dx}\left(18-6a_2+12a_3-12a_2^2+16a_2a_3-28a_2^2a_3\right) \\ +\ 2A\left(12+16a_2-28a_2^2\right)\frac{da_3}{dx} \\ -\ 20\frac{FL}{U}A\,a_2(4a_3+3) - 1680\,\frac{\epsilon}{Re}(1-a_2)\end{array}\right\}}{2A(6+24a_2-16a_3+56a_2a_3)} \tag{23}$$

$$\frac{da_4}{dx} = \frac{\left\{\begin{array}{c} -\frac{3}{56}\frac{dA}{dx} + \frac{3}{Pr\,Re} + \frac{9Ec}{5Re} \\ -\frac{1}{1-\varphi}\frac{\alpha\,Aa_4}{105\,Pr}\left(3\frac{da_2}{dx}+3\frac{da_3}{dx}+7a_2\frac{da_3}{dx}+7a_3\frac{da_2}{dx}\right) \\ -\frac{1}{1-\varphi}\frac{\alpha\,a_4}{420\,Pr}(9+6a_2+6a_3+14a_2a_3)\frac{dA}{dx} - \frac{1}{1-\varphi}\ \frac{2\,\alpha\,\epsilon}{(Pr)^2\,Re}\ a_4 \end{array}\right\}}{\frac{1}{1-\varphi}\ \frac{\alpha A}{210\,Pr}\ (9+6a_2+6a_3+14a_2a_3)} \tag{24}$$

$$\frac{da_3}{dx} = -\ \frac{4a_3+3}{4a_2+3}\ \frac{da_2}{dx} \tag{25}$$

When frictional heat is considered, we use a sixth degree profile

$$\left.\begin{aligned} u &= 1-(1-\ y)^3 \\ u_p &= 1-\ (1-a_2)\,(1-\ y)^3 \\ \rho_p &= 1-\ (1-a_3)\,(1-\ y)^3 \\ T &= \left(1-\tfrac{1}{2}Ec\right)(1-\ y)^3 + \tfrac{1}{2}Ec\,(1-\ y)^6 \\ T_p &= a_4 T \end{aligned}\right\} \tag{26}$$

in the equation (16) , yields to

$$\frac{da_4}{dx} = \frac{-\left(\frac{3}{56}-\frac{9}{560}Ec\right)\frac{dA}{dx}+\frac{3}{PrRe}\left(1+\frac{1}{2}Ec\right)+\frac{9Ec}{5Re} - \frac{1}{1-\varphi}\frac{2\alpha}{3Pr}\left[\frac{1}{2}a_4(A1)\frac{dA}{dx}+Aa_4(A2)\right] - \frac{1}{1-\varphi}\frac{2\alpha\epsilon}{(Pr)^2Re}\left(1+\frac{1}{2}Ec\right)}{\frac{1}{1-\varphi}\frac{2\alpha}{3Pr}A(A1)} \tag{27}$$

Where

$$A1 = -\frac{1}{28} - \frac{3}{280}Ec + \left(\frac{1}{7} - \frac{3}{140}Ec\right)a_2 + \left(\frac{1}{7} - \frac{3}{140}Ec\right)a_3 + \left(\frac{1}{10} - \frac{3}{260}Ec\right)(1-a_2-a_3) + \left(\frac{1}{10} - \frac{3}{260}Ec\right)a_2a_3$$

and

$$A2 = \frac{da_2}{dx}\left\{\frac{3}{70} - \frac{9}{910}Ec + a_3\left(\frac{1}{10} - \frac{3}{260}Ec\right)\right\} + \frac{da_3}{dx}\left\{\frac{3}{70} - \frac{9}{910}Ec + a_2\left(\frac{1}{10} - \frac{3}{260}Ec\right)\right\}$$

A. *Calculation of skin friction* (c_f)

The shearing stress on the plane boundary layer is given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{28}$$

In the present case, using non-dimensional quantities (13),

$$\tau_w = \mu\frac{U}{\delta}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

Using third degree profile,

$$\tau_w = \mu\frac{U}{\delta}\left[\frac{\partial}{\partial y}\{1-(1-y)^3\}\right]_{y=0} = 3\mu\frac{U}{\delta} = \frac{3\rho U^2}{Re_\delta},$$

where $Re_\delta = \frac{\rho U\delta}{\mu}$

and the skin friction coefficient, c_f is given by

$$c_f = \frac{\tau_w}{\rho U^2/2} = \frac{6}{Re_\delta} \qquad (29)$$

B. *Coefficient of Heat Transfer*(Nu)

The coefficient of heat transfer (Nu) is given by

$$Nu_x = \frac{-\left(\frac{\partial T}{\partial y}\right)_{y=0} x}{T_w - T_\infty} \qquad (30)$$

Using non-dimensional quintiles and using the profiles,

$$Nu_x = \begin{cases} \frac{1}{2} Re_x c_f, \text{ without frictional heat} \\ \left(\frac{1}{2} + \frac{1}{4} Ec\right) Re_x c_f, \text{with frictional heat} \end{cases} \qquad (31)$$

III. DISCUSSION OF THE RESULTS

Here in this problem, the basic features like particle velocity, density, temperature, skin friction and heat transfer in the gas particulate boundary layer flow over a flat plate, has been studied by Von Karman - Pohlhausen method.

We choose the following values of the various parameters involved.

$\rho = 0.9752\ kg/m^3$; $\mu = 1.5415 \times 10^{-5}\ kg/m\,sec$; $D = 50\,\mu m,\ 100\,\mu m$; $L = 0.3048\,m$; $U = 60.96\ m/sec$; $\rho_s = 800,\ 2403,\ 8010\ kg/m^3$; $\alpha = 0.1,\ 0.2, 0.3, 0.6$;

Equations (22) to (27) with boundary conditions (17) and (18) are integrated numerically by Runge-Kutta 4th order scheme. The solutions are obtained for different Prandtl number (Pr), volume fraction (φ), material density of SPM (ρ_s), diameter or size of the particle (D), diffusion parameter (ε), concentration parameter (α) for uniform plate temperature. The temperature, velocity and particle density profiles are presented in figures for different values of above parameters. It is seen from Fig. 1 & 2 that the carrier fluid velocity satisfied the no slip condition but the particle velocity profiles do not satisfy no slip condition at the wall and go on increasing with x i.e. towards the downstream of the plate. In Fig. 1 & 4 the profiles for carrier fluid temperature display a simple shape which is found in the thermal boundary layers of pure fluid flow, but the particle temperature on the plate go on increasing with x i.e. towards the downstream of the plate.

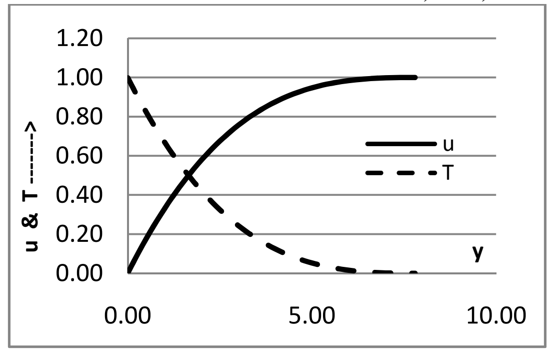

Figure 1: Variation of $\boldsymbol{u}$ and $\boldsymbol{T}$ with $\boldsymbol{y}$

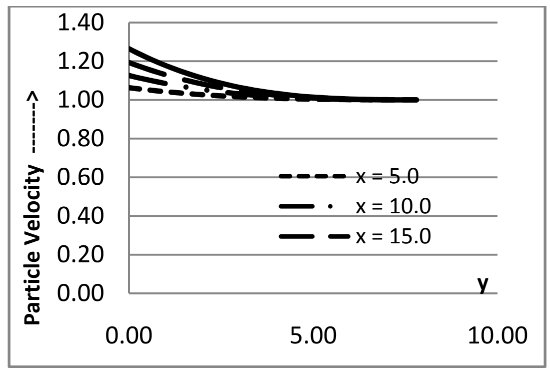

Figure 2: Variation of particle velocity with $\boldsymbol{y}$

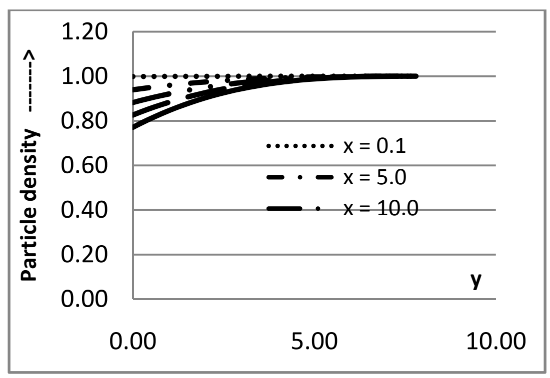

Figure 3: variation of particle density with $\boldsymbol{y}$

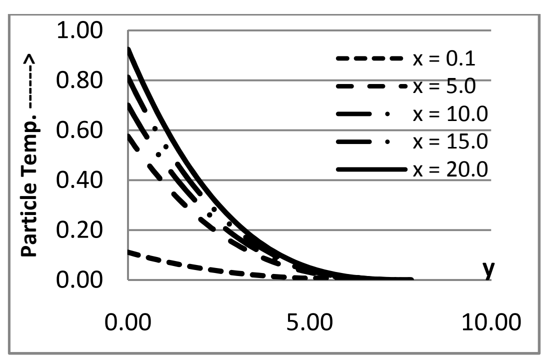

Figure 4: Variation of particle temperature with $\boldsymbol{y}$

Fig. 3 displays the profile for the particle densities, which shows that the density of the particles on the plate goes on decreasing towards the downstream. Table -1 shows that the particle density and particle velocity on the plate assumes a finite value towards the downstream station of the plate.

Table 1: Plate values of velocity, density & temperature for particle phase

x	u_P	ρ_p	T_p
0.10	1.02E+00	9.84E-01	1.05E+00
9.90	2.84E+00	1.03E-01	4.38E+00
19.70	7.30E+00	-2.27E+00	1.19E+01
29.50	2.57E+00	-4.42E+00	2.28E+00
39.30	2.05E+00	-5.12E+00	1.19E+00
49.10	1.98E+00	-5.22E+00	1.05E+00
58.90	1.97E+00	-5.24E+00	1.03E+00
68.70	1.97E+00	-5.24E+00	1.03E+00
78.50	1.97E+00	-5.24E+00	1.03E+00
88.30	1.97E+00	-5.24E+00	1.03E+00
98.10	1.97E+00	-5.24E+00	1.03E+00

From Fig. 5, it is observed that the heavy material particles settle down on the plate, where as the light material particles do not settle on the plate towards far downstream on the plate. In case of the presence of heavier material particles, the skin friction and heat transfer from plate to fluid are more than that of the presence of lighter material particles which is observed in Table 3.

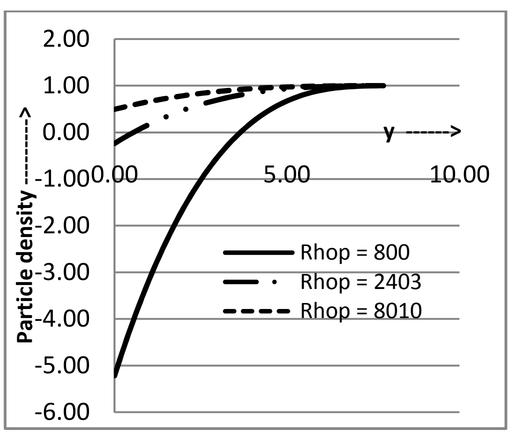

Figure 5 : variation of particle density for different material density of the particle

Fig.6 & 8 shows the presence of coarser particles increase the magnitude of velocity and temperature of the particle phase in comparison with the presence of finer particles inside the boundary layer.

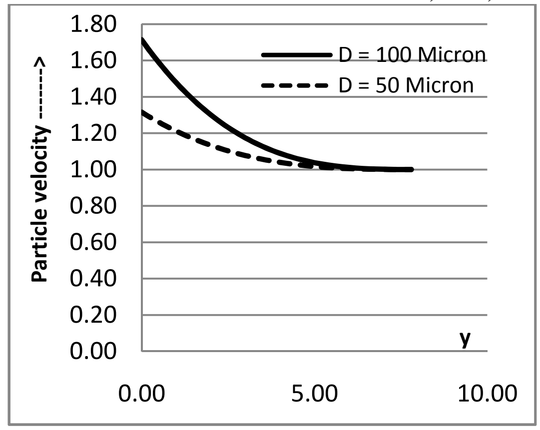

Figure 6: Variation of particle velocity for different size of the particle

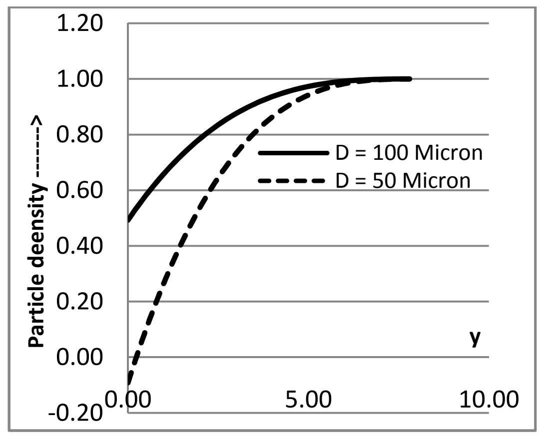

Figure 7: variation of particle density for different size of the particles

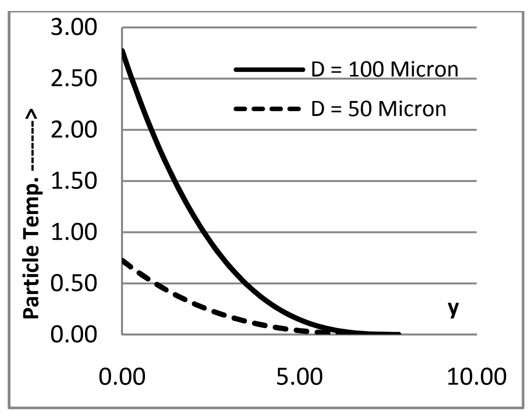

Figure 8: Variation of particle temperature for different size of the particles

The values of Prandtl number (Pr) are taken as 0.71, 1.0 and 7.0 which physically correspond to air, electrolyte solution and water respectively. From Fig. 9, the magnitude of the particle temperature of water is very low as compare to air and electrolyte solution. Fig.10 shows the particle temperature increases as the number of particles per unit volume of the mixture increases.

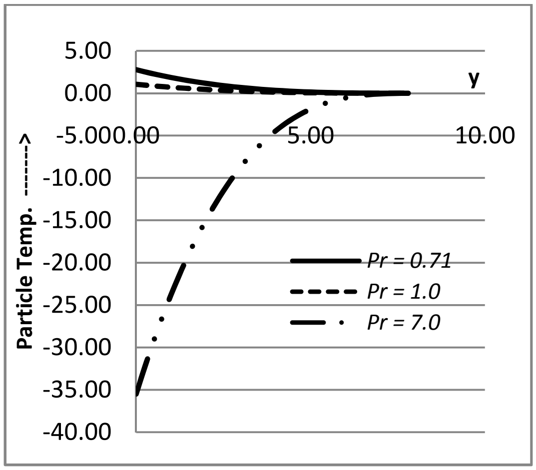

Figure 9: Variation of particle temperature for different $\boldsymbol{Pr}$

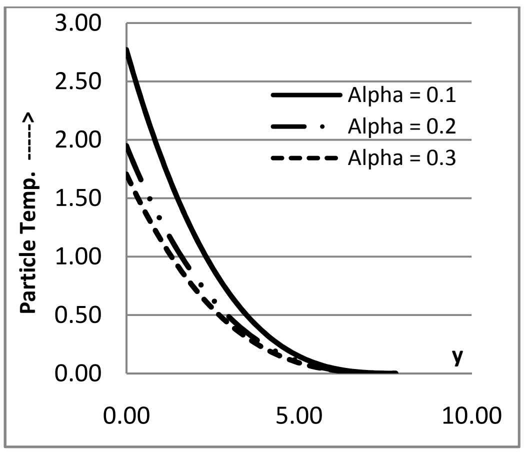

Figure 10: Variation of particle temperature for different $\boldsymbol{\alpha}$

Coarser particles settle more on the plate than that of the finer particles, which can be observed in the Fig.7. Again, from Table 2, skin friction coefficient and heat transfer rate from plate to fluid are more for finer particles.

It is observed from Fig. 11 and 12 that the numerical value of particle temperature with frictional heat $(Ec = 1.0)$ is less than that of without frictional heat$(Ec = 0.0)$. Further, the numerical value of Nusselt number with frictional heat $(Ec = 1.0)$ is greater than that of without frictional heat $(Ec = 0.0)$, indicating that inclusion of frictional heat increases the heat transfer from plate to fluid.

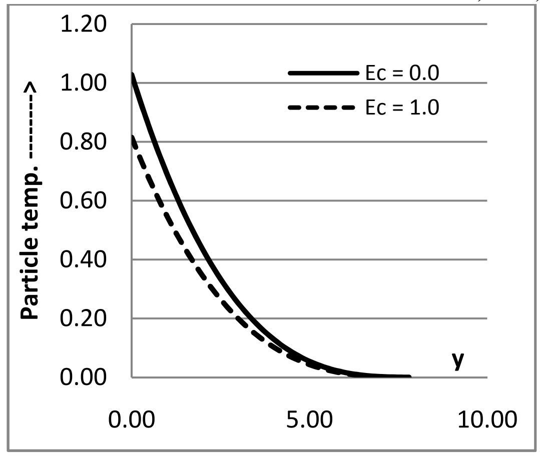

Figure 11: Variation of particle temperature with $\boldsymbol{Ec}$

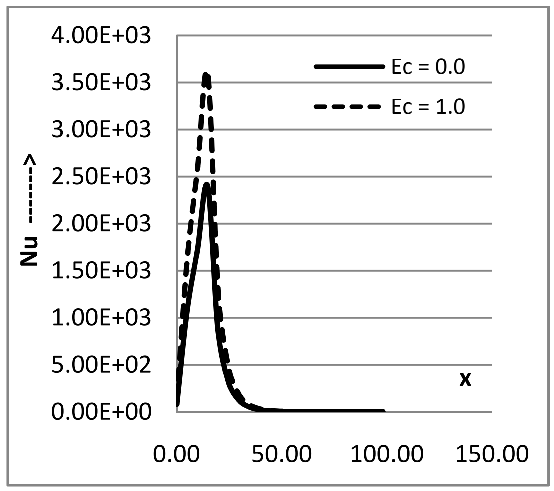

Figure 12: Variation of Nusselt number $(\boldsymbol{Nu})$ with $\boldsymbol{Ec}$

Table 2: Variation of skin friction $(\boldsymbol{c_f})$ & Nusselt number $(\boldsymbol{Nu})$ for different diameter $(\boldsymbol{D})$ / size of the particle

x	skin friction (c_f)		Nusselt number (Nu)	
	$D = 100\mu m$	$D = 50\mu m$	$D = 100\mu m$	$D = 50\mu m$
0.10	9.55E-01	9.00E-01	7.81E+01	7.82E+01
19.70	1.73E-04	1.99E-04	2.00E+03	2.31E+03
39.30	1.28E-04	1.73E-04	2.96E+03	3.99E+03
58.90	1.10E-04	1.41E-04	3.81E+03	4.88E+03
78.50	1.00E-04	1.17E-04	4.62E+03	5.39E+03
98.10	9.41E-05	9.84E-05	5.43E+03	5.67E+03

Table 3: Variation of skin friction $(\boldsymbol{c_f})$ & Nusselt number $(\boldsymbol{Nu})$ for different material density$(\boldsymbol{\rho_s})$ of the particle

	skin friction (c_f)			Nusselt number (Nu)		
x	ρ_s = 800	ρ_s = 2403	ρ_s = 8010	ρ_s =800	ρ_s = 2403	ρ_s = 8010
0.10	6.71E-01	8.18E-01	9.55E-01	7.83E+01	7.82E+01	7.81E+01
19.70	7.45E-05	1.93E-04	1.73E-04	8.62E+02	2.23E+03	2.00E+03
39.30	9.25E-07	1.62E-04	1.28E-04	2.14E+01	3.74E+03	2.96E+03
58.90	1.67E-08	1.49E-04	1.10E-04	5.78E-01	5.15E+03	3.81E+03
78.50	3.04E-10	9.75E-05	1.00E-04	1.40E-02	4.50E+03	4.62E+03
98.10	5.54E-12	8.85E-04	9.41E-05	3.19E-04	5.10E+04	5.43E+03

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