

# Modeling the Image Restoration Performance from Uniform Motion Blur and Poisson Noise

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**Abstract-** Image restoration is to improve the quality of a digital image which has been degraded due to various phenomena like blur, noise. There is an inevitable tradeoff between blur and noise. In order to ease the restoration task, the amount of blur and noise present in a degraded image should be balanced. The effectiveness of all the restoration algorithms depends upon this phenomenon. So to solve this problem we provide a methodology to derive the statistical error model of the restoration algorithm to investigate the performance of any algorithm, considering uniform motion blur and poisson noise. An image with uniform motion blur and Poisson noise acquired at different exposure times are considered. Here PSF is equaled to the exposure time and noise is introduced with mean and standard deviations. These images are analyzed and restored using three different types of algorithms namely LPA-ICI algorithm, Sparse Prior algorithm, Richardson Lucy algorithm based on the error model. In addition to these algorithms, the images are restored using the wavelet transform. The PSNR values of the restored images are calculated. Different methods of restoration show the different PSNR values for the images with the same level of degradation. Hence the performance variance of the restoration algorithms as the blur due to motion develops is shown in the experimental results.

**Index Terms**–Direct Inverse Filter, Uniform motion blur Image restoration, Optimum exposure time.

## I. INTRODUCTION

The blur and the noise are the inter-related concepts when considering an image. Hence the photographers, before acquiring the moving objects or scenes under dim light carefully sets the exposure time, considering the occurrence of blur. But often the captured image is inevitably too blurry or too noisy. Image quality is a characteristic of an image that measures the perceived image degradation typically, compared to an ideal or perfect image. Imaging systems may introduce some amounts of distortion or artifacts; hence the quality assessment becomes an important problem considering exposure time, noise, dynamic range etc. Exposure time is the one of the factor that affects the quality of an image. The exposure time is the time that the shutter is open to capture a scene. Images may be blurred or noisy due to improper or incorrect exposure times. Uniform motion

blur is a blur occurring due to the translational motion between the camera and the scene during the image acquisition process. In recent years uniform motion blur is of interest and research is going on in the area that how does the performance of the restoration techniques varies as the various uniform motion blurred images are considered [1]. The accelerometers in a digital camera cannot sense the uniform motion causing blur and thus a uniform blurred image is obtained [2]. Hence these pictures have to be restored. Early works were considered about the restoration of images under blurred and noisy conditions by comparing the images, with the known point spread functions (PSFs) by means of blur extent and its direction in which it is extended [3]. This method considers about the uniform motion blur and Poisson noise in blind image restoration techniques [4, 5]. Another research proved that the motion blurred images can be restored when several images are given and the direction of the blur in each image is different [7]. Later approach relied on the observations of the derivative filters whose statistics significantly changed due to blur. Thus assuming the blur results from the filters they deblurred the images with the single image assuming constant velocity motion [8]. We are concerned about the restoration of images from uniform motion blur which is a convolution of point spread function (PSF) in the presence of poisson noise.

Direct inverse filter attempts to recover the original image from the observed blurred image. This type of filters leads to significant errors in the restored image, when the  $PSF \approx 0$ . At this stage, noise gets amplified and the restoration becomes inefficient. To avoid these problems a pseudo-inverse filter is used in the existing approach. Many restoration algorithms have been used nowadays to recover the images from the degradation. In the proposed method we have considered the three types of algorithms considering the uniform motion blur and poisson noise as the degradation factor.

The blur and the noise are both governed by a single parameter known as the exposure time. With the incorrect setting of the exposure time, the blur and the noise are inversely proportional to one another. i.e., as the blur increases, noise decreases and as the noise increases, blur decreases. Long exposure time increases the blur and the short exposure time increases the noise of an image [9].

Hence a correct exposure time has to be set. Thus exposure time plays a major role in image acquisition and restoration.

Since the blur and the noise are inter-related, we introduce an image model describing this concept. This method can be applied to raw data for best results. The existing work aims at attaining an optimal exposure time. Optimal exposure time is the time for which the restored image quality can be increased. This can be done using any of the image restoration techniques. This exposure time values may change according to the amount of blur and noise present in an image. For the images that are noise free and blur free, may have the similar optimal exposure time values. In an image restoration process, the optimal exposure time is the time at which the blur and noise present are same amount and thus the quality of an image is increased. Different restoration algorithms may have different optimal exposure times and the degraded images are analyzed using the derived restoration model.

The paper is followed with the existing method in section II, proposed method in section III, experimental results in section IV, conclusion and references in section V and VI respectively.

## II. EXISTING METHOD

### A. Image Model

An image  $z_T$  is modeled with an exposure time of  $T$  as shown below,

$$z_T(x) = k(u_T(x) + \eta(x)) \quad (1)$$

Here  $k$  is the scaling factor and is always  $k > 0$ . This factor is scaled to the signal or image components so as to make the information into a usable allowable range i.e., for normalization  $\eta(x)$  is the poisson noise and  $u_T(x)$  is the uniform motion blur.

#### i. Uniform Motion Blur

The uniform motion blur is modeled here as,

$$\int_{R^2} h_T(s) ds = T \quad (2)$$

i.e., the PSF of a uniform motion blur is equaled to its exposure time in this contribution. A PSF is the phenomenon that explains how long or how the blur is spread over an image. Always a blur has a unit mass. In our model it is assumed to have the value of the exposure time. In general the blurred image is a convolution of an object and its PSF.

#### ii. Poisson noise

The noise is introduced in terms of mean and the standard deviation. Thus it can be re-written as,

$$z_T(x) = E\{z_T(x)\} + std\{z_T(x)\}\alpha(x) \quad (3)$$

Where  $\alpha(x)$  is a random variable, with zero mean and unitary variance. The mean and the standard deviation is given by the equation,

$$E\{z_T(x)\} = kE\{u_T(x)\} \quad (4)$$

$$std\{z_T(x)\} = k\sqrt{\text{var}\{u_T(x)\} + \text{var}\{n(x)\}} \quad (5)$$

#### iii. SNR

The SNR is influenced by the exposure time as the ratio between the mean and the standard deviation of noise. It is given by,

$$SNR(z_T(x)) = \frac{E\{z_T(x)\}}{std\{z_T(x)\}} \cup \sqrt{\lambda T} \rightarrow \alpha \quad (6)$$

The above equation is applicable for large exposure time. The symbol  $\cup$  denotes that both the terms are strictly bounded between the two positive constants,

When  $T$  is small, the SNR is given by,

$$SNR(z_T(x)) = \frac{E\{z_T(x)\}}{std\{z_T(x)\}} \cap T^\beta \rightarrow 0 \quad (7)$$

Where,  $\beta = 0.5$  or  $1$ .

When the blur increases or when the blur is much large enough increasing along with the exposure time, the restoration does not necessarily be accurate. The phenomenon of restoration suits the best for the blur effects due to the uniform motion blur making the SNR ( $Z_T(x)$ ) a meaning factor of the restoration quality. With the increasing blur effects along the increase in exposure time makes the restoration a more challenging factor.

### B. Image Restoration

Diagonal inversion technique is best suited for the image inversion. Here a pseudo-inverse filter is used. A pseudo-inverse filter operates by pixel wise. First the image is divided into several blocks. These blocks are then operated block wise. Each block is restored separately. The appropriate weights are added to the degraded blocks so as to restore them. All the blocks are restored separately by adding the appropriate weights. Finally the blocks are integrated to get the restored images. Thus the images are restored using the pseudo-inverse filter.

The pseudo-inverse filtering method provides acceptable results for restoring degraded images. A threshold value is used for this type of filter and reasonable results can be obtained. In pseudo-inverse filtering images are divided into blocks. Then each pair of pixels is operated by adding the compensating weights to the degraded pixels. Thus the pixels are restored and such operation is carried out for each blocks. Then the images are restored.

The transfer function of a pseudo-inverse filter is given by,

$$\frac{1}{H} = \begin{cases} \frac{1}{H}, & \text{if } H > \varepsilon \\ \varepsilon, & \text{if } H \geq \varepsilon \end{cases} \quad (8)$$

The value of  $\varepsilon$  affects the restored image. With proper selection of  $\varepsilon$ , the restored image quality lies. It is a small positive threshold value (0.05).

### C. Fourier Domain Analysis

Rather than time domain, frequency domain is the domain for the analysis of mathematical functions and signals respect to frequency. Signals represented in Fourier domain also has the information about the phase shift that is applied to each sinusoidal signals so as to recover the original time signals by recombining the frequency components. The image position changes in the frequency domain correspond to the changes in the spatial domain, representing the rate at which image intensity values are changing in the spatial domain image. The Fourier domain transforms the image to its frequency representation, compute inverse transform back to the spatial domain. In this domain, high frequencies correspond to pixel values those changes rapidly across the image (e.g. text, texture, leaves, etc.). Strong low frequency components correspond to large scale features in the image (e.g. a single, homogenous object that dominates the image).

The Fourier domain is chosen here for the purpose of simplicity and to avoid the aliasing effects. The pixels are converted into frequencies and they are analyzed. The SNR on each frequency allows us to speculate on the effects of both blur and noise. The aliasing occurs when a high frequency signal masquerades as a lower frequency signal, this aliasing effects are removed in the Fourier analysis and the optimal SNR and RMSE is obtained.

### D. Optimal Exposure

Restoration is carried out for the degraded images at different exposure times. Restoration gives different quality images at those different exposure times. At one point the noise and the blur are present in the same amount and give the better image quality when restored at that exposure time. This exposure time is said to be the optimum exposure time. Images restored under this exposure time resemble the image quality closely equivalent to the original image.

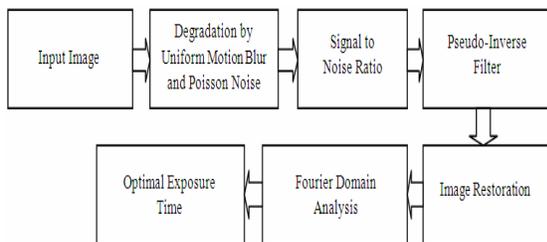


Figure 1. Existing method block Diagram

## III. PROPOSED METHOD

Several restoration algorithms exist to recover the original image from the degraded image. But the performance of these algorithms varies corresponding to the exposure times at which images are captured. In the existing work, the existence of optimal exposure time is shown, at which the amount of blur and noise is balanced. So that the restored image quality can be maximized compared to the other exposure times, considering the uniform motion blur in the poisson noise. In the proposed work we are concerned about the performance variance of the existing restoration algorithms.

The performance or the effectiveness of any algorithm varies depends on the corresponding exposure times. Different algorithms have different optimal exposure times. So in order to ease our work we derive a restoration error model, considering uniform motion blur and poisson noise. The main use of this error model is that, it simultaneously takes into account the exposure times and describes how the performance of the algorithms varies correspondingly. Thus the optimal exposure time can be obtained for any algorithm.

The poisson noise is removed by adding the compensating pixels to the degraded image pixels by adding the sufficient weights to it. Then the three types of deblurring algorithms namely LPA-ICI algorithm, sparse natural image priors and Richardson-Lucy deconvolution algorithm are considered and discussed.

The original clear input image is taken. Uniform motion blur and poisson noise is introduced. The image is degraded at various exposure times at various levels. These images are given to the restoration error model. Then they are restored using the three restoration algorithms namely,

- (1) LPA-ICI algorithm
- (2) Sparse natural image priors
- (3) Richardson-Lucy deconvolution algorithm

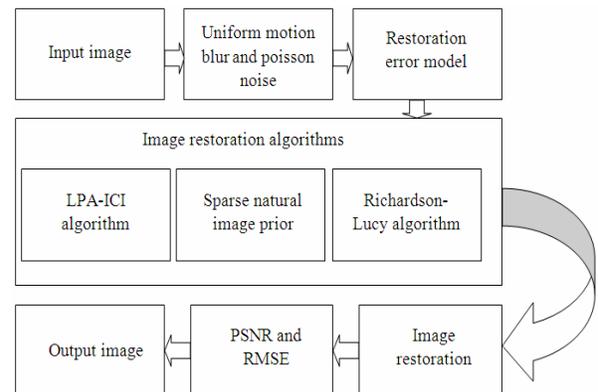


Figure 2. Proposed method block diagram

Finally the PSNR and RMSE values are calculated for the restored image and the output images are obtained. The three different algorithms may have different optimal exposure times.

### A. Restoration Error Model

The effectiveness of the any restoration algorithm typically depends upon the amount of blur and noise present in an image. To face this problem, we model an error model to investigate the different restoration algorithms' performance. The performance of any restoration algorithm depends upon the various factors. One important factor is the exposure time because it balances the amount of noise and blur in a blurred image. Our restoration error model describes how the expected restoration error of a particular image restoration algorithm varies as the blur due to motion develops.

Our proposed methodology can be derived for any restoration algorithm. An error model is derived as the collection of motion PSFs that are representative of blur. These PSFs are considered to be independent.

$$H_T = \{h_T^j\}, \quad j = 1, \dots, m \quad (9)$$

Where,

$H_T$  = Collection of PSF's

$h_T$  = PSF

$j$  = Number of PSF's

Thus the restoration error model is modeled.

Here the collection of PSF's means the collection of uniform motion PSF's obtained at the different exposure times.

The noise model is modeled here as follows,

$$z(x) = y(x) + \sigma(y(x))\xi(x)$$

Where,

$z(x)$  = Noisy image

$y(x)$  = Original image

$\sigma$  = Standard deviation

$$\sigma(y(x))\xi(x) = \eta_p(y(x))$$

where,

$\eta_p$  = Poisson noise component

Poisson noise, is a basic form of uncertainty noise associated with the measurement of light, inherent to the quantized nature of light and the independence of photon detections. Its expected magnitude is signal dependent and constitutes the dominant source of image noise. Removing noise of this type is a more difficult problem.

As the exposure time increases blur also increases and vice versa. i.e., as the exposure time varies the PSF values also vary. Hence the collection of PSF's at various exposure time obtained is said to form the restoration error model. The restoration error is the metric for measuring the deblurring performance, RMSE. While computing these metrics, it is assumed that the images are normalized to a standard intensity range (e.g., [0,255]) in order to get consistent error measurements. The

characteristics of the PSF mostly encompass the influence of the deblurring algorithm. The collection of PSFs can be used to condition the expected error model.

### B. LPA-ICI Algorithm

LPA: Local Polynomial Approximation

ICI: Intersection of Confidence Interval rule.

It is an adaptation algorithm, used to define the most suited neighborhood where the polynomial assumptions fit better the observations. It performs pixel wise polynomial fit on a certain neighborhood. The intersection of confidence intervals (ICI) rule being one of the versions of this approach has appeared to be quite efficient for the adaptive scale image restoration. LPA-ICI is a deconvolution algorithm. It couples the regularized inverse in Fourier domain and the anisotropic filtering in spatial domain.

$$\hat{Y}_{T,\varepsilon}^{RI} = z_T \frac{\overline{kH_T}}{k^2 |H_T|^2 + \varepsilon^2 PSD_T} \quad (10)$$

Where,

RI = Regularized inverse

$H_T$  = Collection of PSFs

PSD = Power Spectral Density of noise

$\varepsilon$  = Regularization parameter

### C. Sparse Prior Deconvolution Algorithm

The use of an exact inverse system can greatly amplify the noise rendering the result useless. In such cases, it is important to utilize prior knowledge regarding the input signal so as to obtain a more accurate estimate of the input signal, even when the system is nearly non-invertible and the observed output signal is noisy. A sparse prior opts to concentrate at a small number of pixels, leaving the majority of image pixels constant. This produces sharper edges, reduces noise and helps to remove unwanted image artifacts such as ringing. To optimize this, we use an iterative re-weighted least squares process. The IRLS approach poses the problem as a sequence of standard least square problems, each least square problem re-weighted by solution at the previous step. The minimization of each least square problem is equivalent to solving a sparse set of image pixels. This algorithm has been successfully applied to raw images.

### D. Richardson-Lucy Deconvolution Algorithm

The Richardson-Lucy algorithm is a well-known iterative method for the deconvolution of images convolved with a known point spread function. It is derived from a statistical point of view as it converges to the maximum-likelihood solution. This assumption holds true for images detected by a digital camera. Here we show an adaption of the Richardson-Lucy algorithm to be used for restoration. Its application to simulated and real data from an imaging radar sensor shows its advantage over the original algorithm.

The Richardson-Lucy deconvolution is an iterative algorithm optimized for Poisson distributed data. Poisson distribution can be assumed in the case that an image is recorded by a digital camera. If convergence is tested for every iteration-step then positive iterative deconvolution is the fastest algorithm. The adapted Richardson-Lucy algorithm outperforms its original as there is no tendency to the noise amplification in areas where the unblurred image function is zero. Richardson-Lucy algorithm, conserves energy, and delivers stable results at high noise-levels. Richardson-Lucy like algorithms led to satisfactory reconstructions. Especially in situations with bad signal-to-noise ratio, the results are very stable.

#### E. Image Restoration Using Wavelet Transform

Image processing techniques like image restoration heavily rely on discrete Fourier transform (DFT) for frequency domain representation for analysis of its frequency content. Wavelet transforms (WT) provide an alternative for the short-time Fourier transform (STFT). Both transforms, STFT and WT result in decomposition of signals into two-dimensional function namely time and frequency. The basic difference between these two transforms falls under the construction of the window function which has a constant length in the case of the STFT while in the WT wide windows are applied for low frequencies and short windows for high frequencies to ensure constant time-frequency resolution. The basic idea of the wavelet transform is to represent the signal to be analyzed as a superposition of wavelets.

First the padding is done for the image pixels in the border. Padding in general is the process of adding unused data to the end of a message in order to make it conform to a certain length. Image padding introduces new pixels around the edges of an image. The border provides space for annotations or acts as a boundary when using advanced filtering techniques. Here the daubechies wavelet function is used to restore the images. The Daubechies wavelets are widely used in solving a broad range of problems. The daubechies wavelet is a family of orthogonal, compactly supported scaling and wavelet functions that have maximum regularity. In wavelet functions the level refers to the number of decomposition steps to perform. The wavelet decomposition results in levels of approximated and detailed information. Single level daubechies decomposition is used in this restoration process. Thus the images are restored using the wavelet transform.

#### IV. EXPERIMENTAL RESULTS

In this section, first the existing method result is shown and then the proposed method results are shown. We show the results of the restoration performance varying with the exposure time. The performance is analyzed by considering a synthetically blurred and noisy observation.

We calculate the root mean square error (RMSE) between the rescaled and the original image by the formula,

$$RMSE = 255 \sqrt{\left(\frac{1}{X}\right) \sum_{x \in X} \left(\frac{1}{k\lambda} y'(x) - y(x)\right)^2} \quad (11)$$

Where,  $y'(x)$  is the restored image and the  $y(x)$  is the original image and  $X$  is the cardinality.

The uniform motion blur and poisson noise is introduced in the different amounts for the input images. We have shown four images with different amounts of degradation. The quality of the restored images varies depending on the degradation. As said before the image quality is governed by the exposure time, thus as shown the degradation of the images increases as the exposure time increases.

Images degraded due to uniform motion blur and poisson noise at various exposure times is shown in the figure 3. The PSNR values are also shown. The sigma value shows how much noise is spread over an image i.e., the noise value. In figure 4, 5, 6, 7 the restored images for LPA-ICI algorithm, Sparse prior algorithm, Richardson-Lucy algorithm and Wavelet transform respectively are shown for the same exposure time. From the result, the image with the better quality has the high PSNR value.

In our proposed method, the three restoration algorithms are considered. Images are degraded with uniform motion blur and poisson noise. The blur is introduced with the phenomena exposure time. As the exposure time increases blur also increases and the noise is introduced with the standard deviation values. Based on the derived error model the images are restored using the algorithms for the same exposure times. The PSNR and the RMSE values are calculated for the restored images. The restored image with high PSNR value is the better restored quality image with high image details and the optimality may vary according to the exposure time for the algorithms.

From the result it is evident that the different algorithms have different PSNR values for the same exposure time. The image with the high PSNR value has the better restoration. Thus it is proved that the optimal exposure time varies from algorithm to algorithm i.e., the performance variance of the different algorithms are clearly shown in the experimental results. In addition to the proposed three algorithms we have compared the results with the restoration performance of the wavelet transform. In the wavelet transform the restoration performance is analyzed using the daubechies wavelet function. The results are obtained using the single level decomposition process.

The results obtained are tabulated in Table 1 and plotted in Figure 8. For the image quality to be high and clear, the PSNR values must be high. It is found that at the different exposure times the performance of wavelet transform is better compared to the other algorithms.

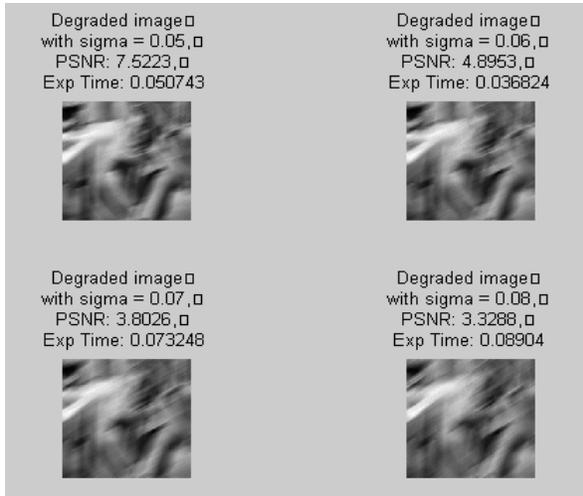


Figure 3. Images degraded due to Uniform Motion Blur and Poisson Noise at various Exposure Time



Figure 4. Images restored using LPA-ICI algorithm



Figure 5. Restored images using Sparse Prior algorithm



Figure 6. Restored images using Richardson-Lucy algorithm



Figure 7. Restored images using Wavelet transform

TABLE I. COMPARISON OF PSNR VALUES

Exposure Time ( sec)	PSNR in dB of Restored Images			
	LPA-ICI Algorithm	Sparse Prior Algorithm	Richardson-Lucy Algorithm	Wavelet Transform
0.050743	44.6564	38.5201	35.4308	55.5752
0.036824	35.6369	36.6872	29.3093	46.5365
0.073248	30.5666	32.6149	33.5773	39.5673
0.08904	27.6567	25.1014	29.4836	37.1993

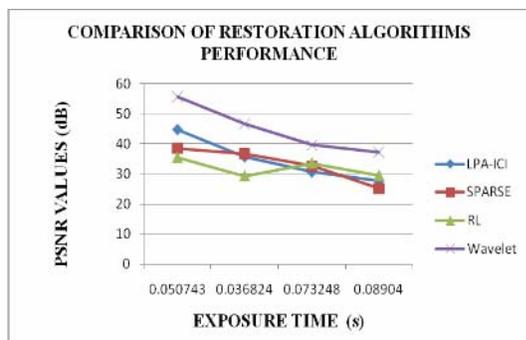


Figure 8. Restoration Performance

## V. CONCLUSION

We have considered the images with uniform motion blur and the poisson noise under various exposure times. A restoration error model is derived, and then the restoration algorithms are used to restore the degraded images. Three algorithms are analyzed based on this error model. In addition to these algorithms restoration performance of the wavelet transform is also performed and it is shown that different restoration methods show different levels of restoration performance at various exposure times.

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