MIXED CONVECTIVE HEAT TRANSFER OF A PARTICULATE SUSPENSION

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***Abstract* - The problem of the two-dimensional, steady and laminar mixed convective boundary layer flow of a fluid with suspended particulate matter(SPM)over a semi-infinite flat plate is solved by momentum integral method. The effect of electrification of particles on the flow and heat transfer has been studied. Numerical calculations were carried out for various values of Prandtl number(Pr), Grashoff number (Gr) , electrification parameter(M), Eckret number(Ec). The electrification of particles enhances the skin friction and heat transfer.**

Key words: Two-phase flow, Boundary layer characteristics, Buoyancy force, Electrification of particles, Shear Stress, Heat transfer.

Nomenclature

$(x, y,)$	Space co-ordinates i.e. distance along the perpendicular to plate length
$\vec{q}(u, v)$	Velocity components for the fluid phase in x and $y-$ directions respectively
$\vec{q}_p(u_p, v_p)$	Velocity components for the particle phase in x and $y-$ directions respectively
(T, T_p)	Temperature of fluid and particle phase respectively
(T_w, T_∞)	Temperature at the wall and free-stream respectively
(υ, υ_p)	Kinematic coefficient of viscosity of fluid and particle phase respectively
(ρ, ρ_p)	Density of fluid and particle phase respectively
(ρ_s, μ_s)	Material density of & viscosity of particles respectively
(μ, μ_p)	Coefficient of viscosity of fluid and particle phase respectively
(τ_p, τ_T)	Velocity and thermal equilibrium time
(c_p, c_s)	Specific heat of fluid and SPM respectively
Re	Fluid phase Reynolds number
Pr	Prandtl number
Ec	Eckret number
Gr	Grashoff Number
Fr	Froud Number
M	Electrification parameter$\left(\frac{FL}{U^2}\left(\frac{e}{m}\right)\right)$
Nu	Nusselt number
c_f	Skin friction coefficient
τ_w	Skin friction (Shear stress for clear fluid)
p	Pressure of fluid phase
φ	Volume fraction of Suspended particulate matter (SPM)
g	Gravitational constant
β^*	Coefficient of volume expansion
D	Diameter of the particle
δ	Boundary layer thickness
a	Thermal diffusivity
κ	Thermal conductivity
α	Concentration parameter
ϵ	Diffusion parameter
F	Friction parameter between the fluid and the particle $\left(F = 18\mu/\rho_p d^2\right)$
L	Reference / Characteristic length
U	Free stream velocity
A	δ^2/L^2
E	Electric field
e	Charge per particle

I. INTRODUCTION

Heat transfer between a particulate suspension and a solid body is a problem whose solution involves the consideration of the equations of motion of a two-phase system. Both Marble[3] and Soo[7] have developed the conservation laws of mass, momentum and energy for two-phase flow. These equations are sufficiently complex to preclude the possibility of exact solutions except in much idealized cases. Most closed form solutions presently known are discussed by Soo[7] and Marble[3].

Singleton[6] has obtained asymptotic solutions using the series expansion method for both small slip region (where the particle slip velocity is small)for dawn stream of the leading edge of the plate and the large slip region (where the slip velocity between two phases is large)close to the leading edge of the plate. He assumed that the density of both phases were not constant. Again He was also assumed that the fluid viscosity is proportional to the square root of its temperature. Wang and Glass [10], considered a moderate slip region (a non equilibrium transition region) in addition to the large- and small- slip regions. They obtained asymptotic series expansion results in all these regions as well as a finite difference solution over the whole plate. Marbel's[3] solutions is valid far down stream region of the plate and the particulate velocity on the surface remains zero. Soo[7] has derived momentum integrals for both the phases and solved the same by using linear profiles both for gas and particle phase velocity, and quadratic profile for particulate density. Soo [7] and Tabakoff and Hammed [8,9] have pointed out that particle velocity decreases linearly along the plate length x and particle density increases continuously along the plate length x. Their study leads to a surface particle velocity zero and particle density to infinity at a distance along the plate length $x = 1$. No effort has been made for studying the temperature distribution inside the boundary layer. Jain & Ghosh [1] have investigated the structure and surface property of the boundary layer of a gas particulate flow over a flat plate by employing momentum integral method. They have pointed out that the third degree profile for velocity and particle density gives results which is valid to far downstream stations on the plate. With the third degree profile of particulate velocity on the surface continuously decreases from its free stream value and particulate density on surface increases rather slowly from its free stream value at the leading edge to an asymptotic value as we approach far downstream on the plate surface. Tripathy & Mishra[5] have investigated the two-phase boundary layer flow over a flat plate to study the boundary layer flow characteristics by using momentum integral method. They have not considered the force due to gravity (Buoyancy force) and electrification of particles in the flow field.

The present paper considers the steady, laminar, incompressible boundary layer flow and heat transfer of a gas-particle suspension over a semi-infinite flat plate. The effect of electrification of Particles and buoyancy force has been considered for better understanding of the boundary layer characteristics and heat transfer.

Particles in air acquire electrical charge by a variety of mechanisms which promote electron transfer to or from the surface, thus producing negatively or positively charged particles, respectively. Soo [7] has studied the effect of electrification on the dynamics of a particulate system. At low temperature, electrification of solid particles occur because of impact with the wall. Even a very slight charge on the solid particles will have a pronounced effect on concentration distribution in the flow of a gas-solid system.

As a general statement, any volume element of charge species, with charge "e" experiences an instantaneous force given by the Lorentz force law,

$$\vec{f} = e\,\vec{E} + \vec{J} \times \vec{B}$$

Where $\vec{B}$ is the magnetic flux density. The current densities in corona discharge are so low that the magnetic force term $\vec{J} \times \vec{B}$ can be omitted, as this term is many orders of magnitude smaller than the Coulomb term $e\vec{E}$.

The ion drift motion arises from the interaction of ions, constantly subject to the Lorentz force with the dense neutral fluid medium. This interaction produces an effective drag force on the ions. The drag force is in equilibrium with the Lorentz force so that the ion velocity in a field $\vec{E}$ is limited to $k_m\vec{E}$, where k_m is the mobility of the ion species. The drag force on the ions has an equal and opposite reaction force acting on the neutral fluid molecules via this ion-neutral molecular interaction, the force on the ions is transmitted directly to the fluid medium, so the force on the fluid particles is also given by above equation.

II. MATHEMATICAL FORMULATION & SOLUTION

Consider the two-dimensional, steady, laminar, two-phase flow that takes place in a half-space bounded by an infinite flat plate. Let the flow be uniform stream parallel to the xy - plane with the plate being coincident with the plane $y = 0$, by choosing the origin of co-ordinates at the leading edge of the plate. The x- axis be along the plate & in a direction opposite to the direction of gravity and y- axis normal to the plate surface, Kaviany[2]. Far away from the plate, both phases are in equilibrium moving with a velocity U in the x- direction. Let the free stream suspension temperature be denoted by T_∞ . Because the plate is infinitely long, the fluid and particle velocity parallel to the plate. Under these considerations, the governing equations of the flow field

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial}{\partial x}(\rho_p u_p) + \frac{\partial}{\partial y}(\rho_p v_p) = 0 \quad (2)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\varphi}{1-\varphi}\frac{\rho_s}{\tau_p}(u - u_p) - \rho g\beta^*(T - T_\infty) + \frac{\varphi}{1-\varphi}\rho_s\left(\frac{e}{m}\right)E \quad (3)$$

$$\varphi\rho_s\left(u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}\right) = \frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right) + \varphi\frac{\rho_s}{\tau_p}(u - u_p) + \varphi(\rho_s - \rho)g + \varphi\rho_s\left(\frac{e}{m}\right)E \quad (4)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\varphi}{1-\varphi}\frac{\rho_s c_s}{\tau_T}(T_p - T) + \frac{\varphi}{1-\varphi}\rho_s\left(\frac{e}{m}\right)Eu_p \quad (5)$$

$$\varphi\rho_s c_s\left(u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y}\right) = \frac{\partial}{\partial y}\left(\varphi\kappa_s\frac{\partial T_p}{\partial y}\right) + \varphi\rho_s c_s\frac{1}{\tau_T}(T - T_p) + \varphi\rho_s\left(\frac{e}{m}\right)Eu_p \quad (6)$$

Subject to the boundary conditions

At $y = 0: u = 0,\ v = 0,\ u_p = a_2(x), v_p = 0,$

$$\rho_p = a_3(x),\ T = T_w,\ T_p = a_4(x) \quad (7)$$

At $y = \delta: u = u_p = U, \rho_p = \rho_{p_\infty},\ T = T_\infty,\ T_p = T_\infty$ (8)

Considering the carrier fluid as incompressible, μ and κ are constant and if the temperature variation is small; μ_s and k_s may be taken as constant. Here the term $\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right)$ may be replaced by $\varphi\mu_s\frac{\partial^2 u_p}{\partial y^2}$, in the particle phase x-momentum equation which arises due to the particle random motion in direct correspondence with similar terms for fluid phase and the term $\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)$ in energy equation for particle phase may be replaced by $\varphi k_s\frac{\partial^2 T_p}{\partial y^2}$. As the free stream velocity U is independent of x, $\frac{\partial p}{\partial x} = 0$.

Clearly $\delta > \delta_t$ and $\delta > \delta_p$

It may be noted that, the thickness of the thermal boundary layer (δ_t), particle velocity boundary layer (δ_p), particle thermal boundary layer (δ_{p_t}) are the same as that of the velocity boundary layer(δ). Strictly speaking, they are different, in general. This assumption has its justification in that it simplifies the computational work and the results obtained are very near to the experimental results and to the exact solutions.

Now, on integration equations (3) to (6) w. r. t. y from $y = 0$ (wall) to $y = \delta$, and by introducing the non-dimensional quantities

$$x^* = \frac{x}{L},\ y^* = \frac{y}{\delta},\ u^* = \frac{u}{U},\ u_p^* = \frac{u_p}{U},\ \rho_p^* = \frac{\rho_p}{\rho_{p_0}},$$

$$T^* = \frac{T - T_\infty}{T_w - T_\infty},\ T_p{}^* = \frac{T_p - T_\infty}{T_{pw} - T_\infty},\ E^* = \frac{E}{E_0} \quad (9)$$

The resulting equations (after dropping stars) are

$$\frac{\partial}{\partial x}\left[\frac{\delta}{L}\int_0^1 u(1-u)\,dy\right] = \frac{\mu}{\rho U\delta}\frac{\partial u}{\partial u}\bigg|_{u=0} + \frac{1}{1-\varphi}F\frac{\rho_{p_0}}{\rho}\frac{\delta}{U}\int_0^1\rho_p(1-u_p)\,dy - \frac{1}{1-\varphi}F\frac{\rho_{p_0}}{\rho}\frac{\delta}{U}\int_0^1\rho_p(1-u)\,dy + \frac{\delta}{L}\int_0^1\frac{Gr}{Re^2}T\,dy - \frac{1}{1-\varphi}\left(\frac{e}{m}\right)\frac{E}{U^2}\frac{\rho_{p_0}}{\rho}\delta\int_0^1\rho_p\,dy \quad (10)$$

$$\frac{\partial}{\partial x}\delta\int_0^1(\rho_p u_p)(1 - u_p)\,dy = \frac{L^2}{\delta}\frac{\epsilon}{Re}\frac{\partial u_p}{\partial y}\bigg|_{y=0} - \delta\frac{FL}{U}\int_0^1\rho_p(u - u_p)\,dy - \frac{\delta}{Fr}\left(1 - \frac{\rho}{\rho_s}\right)\int_0^1\rho_p\,dy - M\delta\int_0^1\rho_p\,dy \quad (11)$$

$$\frac{\partial}{\partial x}\left[\delta\int_0^1 u\,T\,dy\right] = -\frac{aL}{U\delta}\frac{\partial T}{\partial y}\bigg|_{y=0} + \frac{\mu}{\rho c_p}\frac{UL}{\delta(T_w - T_\infty)}\int_0^1\left(\frac{\partial u}{\partial y}\right)^2 dy - \frac{1}{1-\varphi}\frac{c_s}{c_p}\frac{\rho_{p_0}}{\rho}\left(\frac{\partial}{\partial x}\left\{\delta\int_0^1\rho_p u_p T_p dy\right\} - \frac{L^2}{\delta}\frac{\epsilon}{\Pr Re}\frac{\partial T_p}{\partial y}\bigg|_{y=0}\right) \quad (12)$$

and the boundary conditions (7) & (8) reduces to

$y = 0 : u = 0, v = 0, u_p = a_2(x), v_p = 0,$
$\rho_p = a_3(x), T = 1, T_p = a_4(x)$ (13)
$y = 1 : u = u_p = \rho_p = 1, T = 0, \quad T_p = 0$ (14)

For consistency, we use the auxiliary condition that the flux of particulate mass across any control volume is zero.

i.e. $$\rho_{p0}U\delta = \int_0^\delta \rho_p u_p\,dy \quad (15)$$

which gives in non – dimensional form as

$$\frac{d}{dx}\int_0^1\rho_p u_p\,dy = 0 \quad (16)$$

Using the following profiles[1] satisfying the boundary conditions (13,14) as these profiles gives results to far-downstream station on the plate,

$$u = 1 - (1 - y)^3$$

$$u_p = 1 - (1 - a_2)(1 - y)^3$$

$$\rho_p = 1 - (1 - a_3)(1 - y)^3 \quad (17)$$

$$T = (1 - y)^3$$

$$T_p = a_4 (1 - y)^3$$

in the equations (10) to (12), and by suppressing the term due to the frictional heat (2nd term of R.H.S. in eqn. (12)) , we get

$$\frac{dA}{dx} = \frac{56\mu}{\rho U L} - \frac{2}{3}\frac{1}{1-\varphi}\frac{FL}{U}\alpha A a_2 (4a_3 + 3) + \frac{14}{3}\frac{Gr}{Re^2} A - \frac{14}{3}\frac{1}{1-\varphi}\alpha A M (a_3 + 3) \quad (18)$$

$$\frac{da_2}{dx} = \frac{\left\{\begin{array}{c} \frac{dA}{dx}\left(18 - 6a_2 + 12a_3 - 12a_2^2 + 16a_2 a_3 - 28a_2^2 a_3\right) \\ + 2A\left(12 + 16a_2 - 28a_2^2\right)\frac{da_3}{dx} \\ - 20\frac{FL}{U} A\, a_2 (4a_3 + 3) - 1680\frac{\epsilon}{Re}(1 - a_2) \\ + 140\, A \frac{1}{Fr}\left(1 - \frac{\rho}{\rho_S}\right)(a_3 + 3) + 140 M A (a_3 + 3) \end{array}\right\}}{2A(6 + 24a_2 - 16a_3 + 56 a_2 a_3)} \quad (19)$$

$$\frac{da_4}{dx} = \frac{\begin{array}{c} -\frac{3}{56}\frac{dA}{dx} + \frac{3}{Pr\,Re} + \frac{9Ec}{5Re} - \frac{1}{1-\varphi}\frac{\alpha A a_4}{105\,Pr}\left(3\frac{da_2}{dx} + 3\frac{da_3}{dx} + 7a_2\frac{da_3}{dx} + 7a_3\frac{da_2}{dx}\right) \\ - \frac{1}{1-\varphi}\frac{\alpha\, a_4}{420\,Pr}(9 + 6a_2 + 6a_3 + 14a_2 a_3)\frac{dA}{dx} - \frac{1}{1-\varphi}\frac{2\,\alpha\,\epsilon}{(Pr)^2\,Re}\, a_4 \end{array}}{\frac{1}{1-\varphi}\frac{\alpha A}{210\,Pr}(9 + 6a_2 + 6a_3 + 14a_2 a_3)} \quad (20)$$

$$\frac{da_3}{dx} = -\frac{4a_3 + 3}{4a_2 + 3}\frac{da_2}{dx} \quad (21)$$

When frictional heat is considered, we use a sixth degree profile

$$u = 1 - (1 - y)^3$$

$$u_p = 1 - (1 - a_2)(1 - y)^3$$

$$\rho_p = 1 - (1 - a_3)(1 - y)^3 \quad (22)$$

$$T = \left(1 - \frac{1}{2}Ec\right)(1 - y)^3 + \frac{1}{2}Ec\,(1 - y)^6$$

$$T_p = a_4 T$$

in the equation (12) , yields to

$$\frac{da_4}{dx} = \frac{\begin{array}{c} -\left(\frac{3}{56} - \frac{9}{560}Ec\right)\frac{dA}{dx} + \frac{3}{PrRe}\left(1 + \frac{1}{2}Ec\right) + \frac{9Ec}{5Re} \\ - \frac{1}{1-\varphi}\frac{2\alpha}{3Pr}\left[\frac{1}{2}a_4(B_1)\frac{dA}{dx} + A a_4 (B_2)\right] - \frac{1}{1-\varphi}\frac{2\alpha\epsilon}{(Pr)^2 Re}\left(1 + \frac{1}{2}Ec\right) \end{array}}{\frac{1}{1-\varphi}\frac{2\alpha}{3Pr}A(AP)} \quad (23)$$

Where

$$B_1 = -\frac{1}{28} - \frac{3}{280}Ec + \left(\frac{1}{7} - \frac{3}{140}Ec\right)a_2 + \left(\frac{1}{7} - \frac{3}{140}Ec\right)a_3 + \left(\frac{1}{10} - \frac{3}{260}Ec\right)(1 - a_2 - a_3) + \left(\frac{1}{10} - \frac{3}{260}Ec\right)a_2 a_3$$

and

$$B_2 = \frac{da_2}{dx}\left\{\frac{3}{70} - \frac{9}{910}Ec + a_3\left(\frac{1}{10} - \frac{3}{260}Ec\right)\right\} + \frac{da_3}{dx}\left\{\frac{3}{70} - \frac{9}{910}Ec + a_2\left(\frac{1}{10} - \frac{3}{260}Ec\right)\right\}$$

Calculation of Skin friction (c_f)

The shearing stress on the plane boundary layer is given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (24)$$

In the present case, using non-dimensional quantities (9),

$$\tau_w = \mu\frac{U}{\delta}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

Using third degree profile,

$$\tau_w = \mu\frac{U}{\delta}\left[\frac{\partial}{\partial y}\{1 - (1 - y)^3\}\right]_{y=0} = 3\mu\frac{U}{\delta} = \frac{3\rho U^2}{Re_\delta},$$

where $Re_\delta = \frac{\rho U \delta}{\mu}$

and the skin friction coefficient, c_f is given by

$$c_f = \frac{\tau_w}{\rho U^2/2} = \frac{6}{Re_\delta} \quad (25)$$

Calculation of Coefficient of Heat Transfer (Nu)

The coefficient of heat transfer (Nu) is given by

$$Nu_x = \frac{-\left(\frac{\partial T}{\partial y}\right)_{y=0} x}{T_w - T_\infty} \quad (26)$$

Using non-dimensional quintiles and using the profiles,

$$Nu_x = \begin{cases} \frac{1}{2}Re_x c_f, \text{when frictional heat is not considered} \\ \left(\frac{1}{2} + \frac{1}{4}Ec\right)Re_x c_f, \text{when frictional heat is considered} \end{cases} \quad (27)$$

III. DISCUSSION OF THE RESULTS

Here in this problem, the basic features like particle velocity, density, temperature, skin friction and heat transfer in the gas particulate boundary layer flow over a flat plate, has been studied by Von Karman - Pohlhausen method.

We choose the following values of the various parameters involved.

$\rho = 0.9752\ \ kg/m^3$; $\mu = 1.5415 \times 10^{-5}\ kg/m\ s$; $L = 0.3048\,m$; $D = 50\,\mu m, 100\,\mu m$; $\rho_s = 800\,kg/m^3$; $\alpha = 0.1$; $\epsilon = 0.05$; $U = 60.96\,, 160.96\,m/s$; $g = 9.8\,m/s^2$; $Ec = 0.0, 1.0$; $Pr = 0.71, 1.0, 7.0$

Runge-Kutta 4th order scheme has been employed to integrate equations (18) to (21) and (23) for different values of Prandtl number(Pr), Grashoff number (Gr) , electrification parameter(M), Eckret number (Ec), diameter or size of the particle (D), diffusion parameter (ε), concentration parameter (α) for uniform plate temperature.

It is observed from Fig. 1 & 2 that the carrier fluid velocity satisfies the no slip condition but the particle velocity profiles does not satisfies no slip condition at the wall and go on decreasing with x i.e. towards the downstream of the plate. In Fig. 1 & 4 the profiles for carrier fluid temperature display a simple shape which is found in the thermal boundary layers of pure fluid flow, but the particle temperature on the plate go on increasing with x i.e. towards the downstream of the plate. Fig. 3 displays the profile for the particle densities, which shows that the density of the particles on the plate goes on decreasing towards the downstream. Fig. 2 & 3 clearly shows that, the particle velocity and particle density on the plate assumes a finite value towards the downstream station of the plate.

Fig.5 shows that the coarser particles move faster than that of finer particles. The particle temperature becomes negative in case of coarser particles indicating the particles are hotter than the fluid and heat flows from particles to fluid, as shown in the Fig. 6.

The values of Prandtl number (Pr) are taken as 0.71, 1.0 and 7.0 which physically correspond to air, electrolyte solution and water respectively. From Fig. 7, the magnitude of the particle temperature of water is very low as compare to air and electrolyte solution.

It is observed from Fig. 8 and 9 that the numerical value of particle velocity and temperature with frictional heat $(Ec = 1.0)$ is less than that of without frictional heat$(Ec = 0.0)$. Further, the numerical value of Nusselt number with frictional heat$(Ec = 1.0)$ is greater than that of without frictional heat $(Ec = 0.0)$, indicating that inclusion of frictional heat increases the heat transfer from plate to fluid, which is indicates in Table - 1. Further it can be observed that, Nusselt number(Nu) increases with M, the electrification parameter, either in case of with or without frictional heat.

Table 2 & 3 shows that, both skin friction(c_f) and Nusselt number (Nu) increases with M indicating electrification of particles increases the heat transfer.

From Fig. 10 and 11, it can be observed that, the magnitude of particle velocity and temperature increases with the increase of M, the electrification parameter.

CONCLUSION

Fourth order Runge-Kutta method has been employed to obtain the numerical results. From the figures and tables, we have observed that electrification of particles enhance the skin friction as well as the Nusselt number indicating increasing in heat transfer from plate to fluid. The particle phase temperature becomes negative in case of the presence of coarser particles, indicating the particles hotter than the fluid and heat transfer is from particles to fluid.

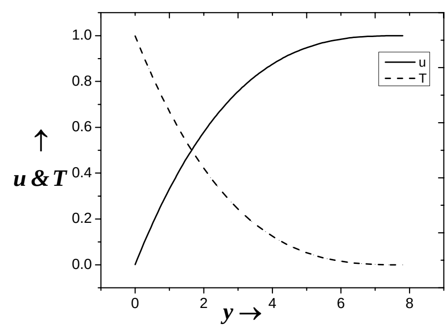

Fig. 1: Variation of carrier fluid velocity & temperature

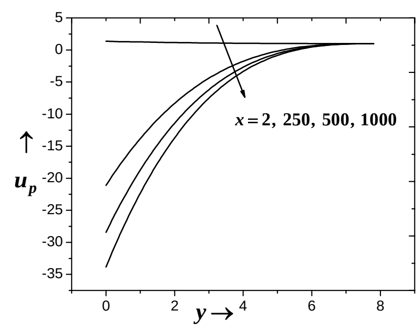

Fig. 2: Variation of particle velocity with y at different downstream stations(x)

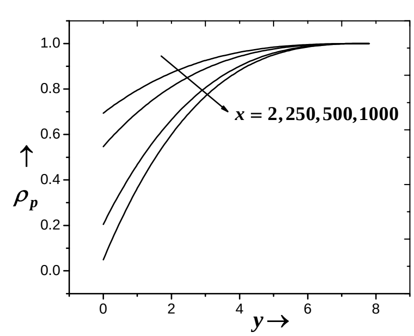

Fig. 3: Variation of particle density with y at different downstream stations(x)

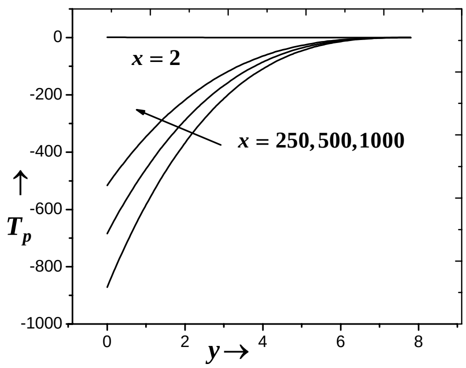

Fig. 4: variation of particle temperature with y at different downstream stations(x)

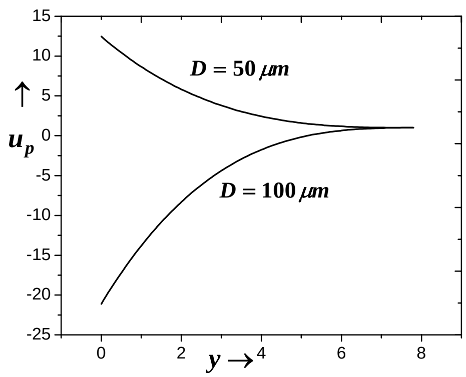

Fig. 5: Variation of particle velocity with y for different size of particles (D)

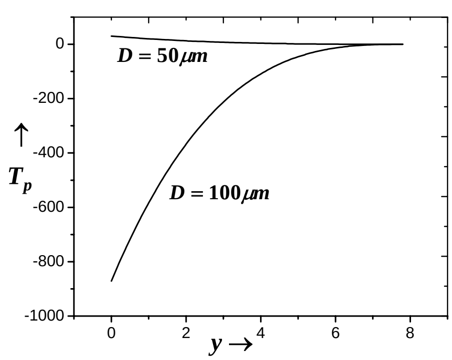

Fig. 6: Variation of particle temperature with y for different size of particles(D)

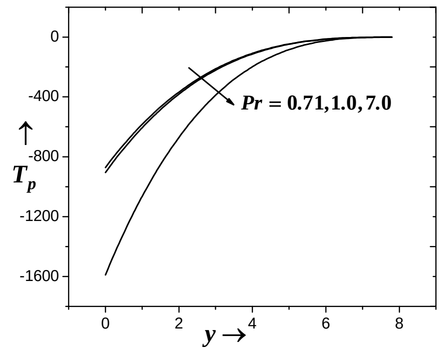

Fig. 7: Variation of particle temperature with y for different Prandtl number (Pr)

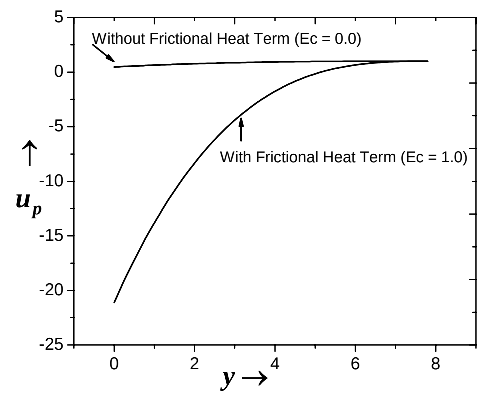

Fig. 8: Variation of particle velocity with y for different Ec.

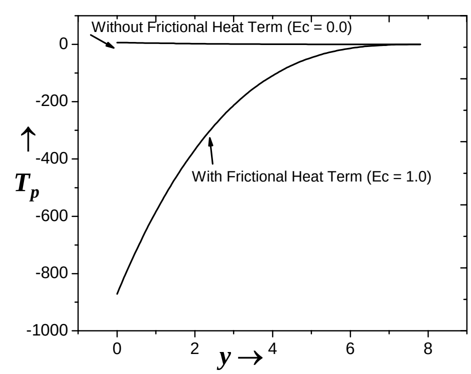

Fig. 9: Variation of particle temperature with y for different Eckret number (Ec).

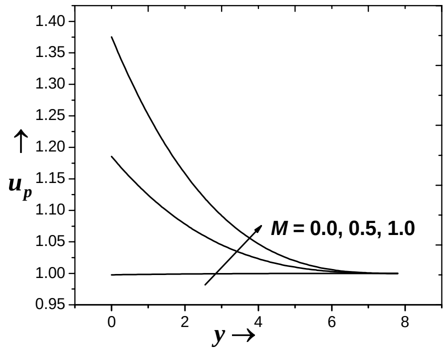

Fig. 10: Variation of particle velocity with y for different electrification parameter(M).

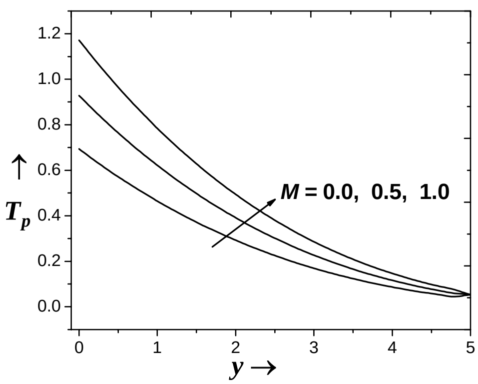

Fig. 11: Variation of particle temperature with y for different electrification parameter(M).

Table 1: Variation of Nusselt Number (Nu) with x for different values of Ec and M

x	$Ec = 0$, $M = 0$	$Ec = 0$, $M = 1$	$Ec = 1$, $M = 0$	$Ec = 1$, $M = 1$
0.10	7.53E+01	8.12E+01	1.26E+02	1.37E+02
5.00	2.17E+02	8.50E+02	4.31E+02	4.02E+03
9.90	5.12E+01	2.93E+02	9.27E+01	7.22E+03
19.70	1.46E+00	5.76E+00	2.19E+00	1.20E+04
29.50	3.14E-02	8.52E-02	3.90E-02	1.53E+04
39.30	6.00E-04	1.12E-03	6.18E-04	1.73E+04
49.10	1.07E-05	1.38E-05	9.17E-06	1.84E+04
58.90	1.85E-07	1.64E-07	1.31E-07	1.87E+04
68.70	3.09E-09	1.89E-09	1.81E-09	1.84E+04
78.50	5.07E-11	2.13E-11	2.46E-11	1.77E+04
88.30	8.17E-13	2.37E-13	3.29E-13	1.68E+04
98.10	1.30E-14	2.60E-15	4.35E-15	1.57E+04

Table 2: Variation of Nusselt number (Nu) along the plate with different electrification parameter (M)

x	$M = 0.0$	$M = 0.1$	$M = 1.0$
0.10	1.26E+02	1.27E+02	1.37E+02
5.00	4.31E+02	6.25E+02	4.02E+03
9.90	9.27E+01	2.04E+02	7.22E+03
19.70	2.19E+00	1.11E+01	1.20E+04
29.50	3.90E-02	4.59E-01	1.53E+04
39.30	6.18E-04	1.68E-02	1.73E+04
49.10	9.17E-06	5.77E-04	1.84E+04
58.90	1.31E-07	1.90E-05	1.87E+04
68.70	1.81E-09	6.10E-07	1.84E+04
78.50	2.46E-11	1.92E-08	1.77E+04
88.30	3.29E-13	5.93E-10	1.68E+04
98.10	4.35E-15	1.81E-11	1.57E+04

Table 3: Variation of Skin Friction (c_f) along the plate with different electrification parameter (M)

x	$M = 0.0$	$M = 0.1$	$M = 1.0$
0.10	1.01E+00	9.72E-01	9.50E-01
5.00	3.70E-05	5.37E-05	3.45E-04
9.90	4.02E-06	8.84E-06	3.13E-04
19.70	4.78E-08	2.43E-07	2.62E-04
29.50	5.68E-10	6.68E-09	2.22E-04
39.30	6.75E-12	1.84E-10	1.89E-04
49.10	8.02E-14	5.05E-12	1.61E-04
58.90	9.54E-16	1.39E-13	1.36E-04
68.70	1.13E-17	3.82E-15	1.15E-04
78.50	1.35E-19	1.05E-16	9.70E-05
88.30	1.60E-21	2.88E-18	8.16E-05
98.10	1.90E-23	7.93E-20	6.86E-05

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