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A Novel approach to Minimum Cost flow of Fuzzy Assignment Problem with Fuzzy Membership Functions

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***Abstract*— In this paper, we propose a new algorithm using the Robust's ranking method to find the optimal fuzzy assignment cost for the fuzzy assignment problem in which the parameters are Trapezoidal Fuzzy Numbers. A numerical example is included.**

Key Words : Assignment Problem; α - cut membership function, Trapezoidal Fuzzy Numer.

I. Introduction:

An assignment problem plays an important role in industry and other applications. It is completely specified by its two components viz, the assignments which represent the underlying combinatorial structure, and the objective function to be optimized which models "the best possible way". The assignment problem refers to another special class of linear programming problem where the objective is to assign a number of resources to an equal number of activities on a one to one basis so as to minimize total costs of performing the tasks at hand or maximize total profit of allocation.

In other words, the problem is, how should the assignment be made so as to optimize the given objective. Different methods have been presented for assignment problem and various articles have been published in[1], [2] and [3] . However, in many real-world applications, job costs are not deterministic. Fuzzy theory has been supported to slash the risk of improper models and solutions which do not reflect the practical problems **[7].** Many researchers have stated that the Assignment problem and its variants under the fuzzy environments. Lin and Wen **[4]** proposed an efficient algorithm based on the labeling method for solving the linear fractional programming case. Longsheng Huang and Li-pu Zhang [5] developed a mathematical model for the fuzzy assignment problem and transformed the model as certain assignment problem with restriction of qualification. Chen Liang-Hsuan and Lu Hai-Wen [3] developed a procedure for resolving assignments problem with multiple inadequate inputs and outputs in crisp form for each possible assignment using linear programming model. To determine the assignments with the maximum efficiency. Majumdar and Bhunia [7] developed an exclusive genetic algorithm to solve a generalized assignment problem with imprecise cost(s)/time(s). Jiuping Xu [9] developed a priority based genetic algorithm to a fuzzy vehicle routing assignment model with connection network. In [1,2] described an efficient fuzzy optimal solution of the assignment problem is based on the ranking function and alpha level membership functions. [8] Proposed a ranking of fuzzy cost present in the fuzzy assignment problem, which takes more advantages over the existing fuzzy ranking methods. Here we propose a new algorithm using the Robust's ranking method to find the optimal fuzzy assignment cost for the fuzzy assignment problem in which the parameters are Trapezoidal Fuzzy Numbers.

2 PRELIMINARIES

Zadeh.L [11] in 1965 first introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life. It is used in all branches of science and engineering.

2.1 Definition:[11]

A Fuzzy set is characterized by a membership function mapping elements of a domain ,space or universe of discourse X to the unit interval[0,1].(i.e) $A = \{(x, \mu_A(x)), x \in X\}$. Here $\mu_A : X \to [0,1]$ is a mapping of the degree of the membership function of the fuzzy set A and $x \in X$ is called the membership value of $x \in X$ in fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2.2 Definition:[11]

A fuzzy set A of the universe of discourse X is called a normal fuzzy set exists at least one $x \in X$ such that $\mu_A(x) = 1$. The fuzzy set A is convex if and only if for any $x_1, x_2 \in X$, the membership function of A satisfies the inequality $\mu_A\{\lambda x_1 + (1-\lambda)x_2\}$.(i.e) min $\{\mu_A(x_1), \mu_A(x_2)\}, 0 \le \lambda \le 1$.

2.3 Definition:[11]

The α - cut of a fuzzy number A(x) is defined as: $A(\alpha) = \{x / \mu_A(x) \ge \alpha, \alpha \in [0,1]\}$.

2.4 Definition: [9]

For a trapezoidal fuzzy number A(x) can be represented by A(a,b,c,d) with membership function $\mu_A(x)$ is given by

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{d-x}{d-c}, & c \le x \le d \end{cases}$$

2.5 Definition:[6]

Let A be a trapezoidal fuzzy number, then the Robust ranking function is defined R(a) as follows: $R(a) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U) d\alpha$, Where (a_α^L, a_α^U) is the α- level cut of the fuzzy number A

3 MATHEMATICAL FORMULATION OF ASSIGNMENT PROBLEM

The assignment problem is a special case of transportation problem in which the objective is to assign 'm' jobs or workers to 'n' machines such that the cost incurred is minimized. The cost of the assignment problem is same as that of a Transportation problem except that availability at each of the resource and the requirement at each of the destinations is unity. Let x_{ij} denote the assignment of i[th] resource to j[th] activity, such that

$$x_{ij} = \begin{Bmatrix} 1 \text{ if resource } i \text{ is assigned with the } j \\ 0 \text{ otherwise} \end{Bmatrix}$$

Then the mathematical formulation of the assignment problems is

Minimum $z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$ Subject to $\sum_{i=1}^{n} x_{ij} = 1$, $\sum_{j=1}^{n} x_{ij} = 1$

The assignment model can be solved directly as a regular transportation model. The fact that all the supply and demand amounts equal 1 has led to the development of a simple solution algorithm called the Hungarian method. In Most of time the decision maker decision is uncertain. So, let us consider the $\overline{c_{ij}}$ be the fuzzy cost of the i[th] person is assigned to the j[th] job. The mathematical formulation of the fuzzy assignment problem as follows: Let $\overline{x_{ij}}$ denote the assignment of i[th] resource to j[th] activity, such that

$$\overline{x_{ij}} = \begin{Bmatrix} 1 \text{ if resource } i \text{ is assigned with the } j \\ 0 \text{ otherwise} \end{Bmatrix}$$

Then the mathematical formulation of the Fuzzy assignment problems as follows:

Minimimum $\overline{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \overline{c_{ij}}\,\overline{x_{ij}}$ Subject to $\sum_{i=1}^{n} \overline{x_{ij}} = 1$, $\sum_{j=1}^{n} \overline{x_{ij}} = 1$,

where $\overline{x_{ij}} = [x_{ij}^L, x_{ij}, x_{ij}^U]$ denotes the fuzzy variables, $\overline{c_{ij}} = [c_{ij}^L, c_{ij}, c_{ij}^U]$ denotes the fuzzy cost

4 PROPOSED SOLUTION METHODOLOGY

We proposed a new algorithm to determine the Minimal Cost flow of the Fuzzy Assignment Problem.

Step 1: Construct the Fuzzy Assignment Table.

Step 2: Using Rank function, the given FAP (step(i)) is Converted to AP

Step 3: Select the Minimum odd cost from all cost of the reduced table (step (2))

Step 4: Subtract selected least odd cost only from the odd cost of the (step (3)). Now, there will be at least one zero and remaining all cost become even.

Step 5: Divide by 2 each column from the step (4)

Step 6: Repeat step 3 and Step 4.

Step 7 : If it allocate one assignment for one cell , then the optimality condition exists.

Step 8: Examine the columns successively until a column with exactly one marked zero is found. Enclose this zero with a circle and an assignment is made to that cell. Mark a cross in the cells of all other zeros lying in the row of enclosed zero. Continue this process until all the columns have been considered.

Step 9: Repeat step 8 successively until one of the following two cases arises:

Case (i): No unmarked zero is left STOP.

Case (ii): There are more than one unmarked zeros arbitrarily and mark a cross in the cells of remaining zeros in its row and column. Repeat the process until no unmarked zeros arbitrarily and mark a cross in the cells of remaining zeros in its row and column. Repeat the process until no unmarked zero is left in the cost matrix.

Step10: There is exactly one encircled zero in each row and each column of the cost matrix. The assignment schedules corresponding to these zeros in the optimum assignment then replace C_{ij} by R (C_{ij}).

4.1 Numerical Example

A National Truck – Rental service has a surplus of one truck in each of the cities 1,2,3,4 and a deficit of one truck in each of the cities A, B, C, D. The distances between (in Km) the cities with surplus and the cities with deficit are displayed, the distances are considered as trapezoidal fuzzy number as below:

Machine /Job	A	B	C	D
1	(4,6,8,10)	(20,24,26,28)	(14,16,17,19)	(6,8,11,13)
2	(8,11,13,15)	(24,26,28,30)	(0,2,4,6)	(23,25,26,28)
3	(30,34,38,40)	(15,17,19,30)	(14,16,18,20)	(12,14,15,18)
4	(14,16,19,20)	(22,24,26,28)	(20,22,24,26)	(6,8,10,12)

How should the trucks by dispersed so as to minimize the total distance travelled?

Solution: It is a balanced assignment problem. Using proposed algorithm, each machine has to assign each job. The optimality condition is satisfied.

Machine/Job	A	B	C	D
1	0	3	2	0
2	1	3	0	1
3	4	0	0	0
4	0	1	2	0

The Optimal Fuzzy Assignment table is:

Machine /Job	A	B	C	D
1	(4,6,8,10)	(20,24,26,28)	(14,16,17,19)	**(6,8,11,13)**
2	(8,11,13,15)	(24,26,28,30)	**(0,2,4,6)**	(23,25,26,28)
3	(30,34,38,40)	**(15,17,19,30)**	(14,16,18,20)	(12,14,15,18)
4	**(14,16,19,20)**	(22,24,26,28)	(20,22,24,26)	(6,8,10,12)

The Fuzzy optimal Assignment is :

A $\rightarrow$4, B $\rightarrow$3, C $\rightarrow$2, D$\rightarrow$1

$\overline{z_{ij}}$ =(6,8,11,13) + (0,2,4,6) + (15,17,19,30) +(14,16,19,20)

$R(\overline{z_{ij}})$ = (35, 51, 53, 69)

Determination of α - cut of Optimal Fuzzy Assignment Cost :

α - cut Fuzzy Assignment values are computed from their membership functions as follows:

$$\mu_{C14}(x) = \begin{cases} \dfrac{x-6}{8-6} & 6 \le x \le 8 \\ \dfrac{13-x}{13-11} & 11 \le x \le 13 \end{cases}$$

$$C_{14}^{(\alpha)} = [2\alpha + 6, 13 - 2\alpha] \text{ ---------- (A)}$$

Similarly ,
$$C_{23}^{(\alpha)} = [2\alpha, 6 - 2\alpha] \text{ ---------------- (B)}$$
$$C_{33}^{(\alpha)} = [2\alpha + 15, 30 - 11\alpha] \text{ --------- (C)}$$
$$C_{41}^{(\alpha)} = [2\alpha + 14, 20 - \alpha] \text{ ----------- (D)}$$

α - Cut of the Optimal Fuzzy Assignment value is

$$Z^{(\alpha)} = [8\alpha + 35, 69 - 16\alpha]$$

The varing of fuzzy transportation cost due to the degree of certainty is illustrated in the table:

α- Degree of Certain	Rang of the Optimal Fuzzy Assignment value using α- cut value of the FAP	Robust Ranking Technique
0	[35, 69]	52
0.1	[35.8, 67.4]	51.6
0.2	[36.6, 65.8]	51.2
0.3	[37.4, 64.2]	50.8
0.4	[38.2, 62.6]	50.4
0.5	[39, 61]	50
0.6	[39.8, 59.4]	49.6
0.7	[40.6, 57.8]	49.2
0.8	[41.4, 56.2]	48.8
0.9	[42.2, 54.6]	48.4
1.0	[43, 53]	48

Weighted Interval Assignment Value is:

$W(z_{ij})$ = (0 x 52) + (.1 x 51.6) + (.2 x 51.2) + (.3 x 50.8) + (.4 x 50.4) + (.5 x 50) + (.6 x 49.6) + (.7 x 49.2) + (.8 x 48.8) + (.9 x 48.4) + (1 x 48)

$W(z_{ij})$ = 5.16 + 10.24 + 15.24 + 20.16 + 25 + 29.76 + 34.44 + 39.04 + 43.56 +48

$W(z_{ij})$ = (270.6/ 10) = 27.06

The average Minimum Fuzzy Assignment Value is :

(550 / 11) = 50

5 Conclusion

Assignment problem is one of the most important problems in decision making. In many real life applications, costs of AP are not deterministic numbers. The FAP is more realistic than the AP because most real environments are uncertain. In recent years, many researchers investigate AP and its variants under fuzzy environments. The proposed method namely; zero point method may be an effective tool to obtain the Optimal Fuzzy Assignment Cost of FAP.

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